Minimum-Entropy, PDF Approximation, and Kernel Selection for Measurement Estimation

José Ismael De la Rosa, Gilles A. Fleury, and Marie-Eve Davoust

***Abstract*—The purpose of this paper is to investigate the selection of an appropriate kernel to be used in a recent robust approach called minimum-entropy estimator (MEE). This MEE estimator is extended to measurement estimation and pdf approximation when $\wp(\mathrm{e})$ is unknown. The entropy criterion is constructed on the basis of a symmetrized kernel estimate $\hat{\wp}_{n,h}(\mathrm{e})$ of $\wp(\mathrm{e})$. The MEE performance is generally better than the Maximum Likelihood (ML) estimator. The bandwidth selection procedure is a crucial task to assure consistency of kernel estimates. Moreover, recent proposed Hilbert kernels avoid the use of bandwidth, improving the consistency of the kernel estimate. A comparison between results obtained with normal, cosine and Hilbert kernels is presented.**

***Index Terms*—Bootstrap, indirect measurement, Monte Carlo simulation, nonlinear regression, nonparametric PDF estimation, robust estimation.**

I. Introduction

In many industrial applications, direct access to a measurement ($\boldsymbol{m}$) is not possible. Yet, as an estimation of the measurement is needed, the process must be considered as an inverse problem [1]. The problem of probability density function (pdf) estimation for such indirect measurement $\wp(\boldsymbol{m})$ is considered in this work. The uncertainty characterization of the measurement has been introduced for a nonlinear Gaussian framework [2]. However, sometimes the errors pdf is unknown [3], [4] or even non-Gaussian, such as uniform, approximately uniform, Laplace, mixture, etc. The errors are still considered as independent identically distributed (i.i.d.) samples drawn from $\wp(\boldsymbol{e})$. Nonparametric methods have shown to be attractive tools for density estimation. The kernel density estimation is another class of such methods. A recent work presented by Pronzato and Thierry [5], [6] shows the applicability of kernel density estimation in regression problems, they propose a minimum-entropy estimator, which seems to give very good performance in both Gaussian and in non-Gaussian error assumptions. These ideas are extended in this work to the measurement estimation problem.

A very good introduction to the kernel density estimation is given in different works [7]–[10]. Some of these works are dedicated to bandwidth h selection methods; in fact, the bandwidth selection is a crucial task to ensure the consistency of the density estimates via kernel estimation (controling the levels of smoothing—oversmoothing/undersmoothing). On the other hand, cosine-based weight functions replace h by a parameter q, for which computation is more efficient. Finally, Hilbert kernels avoid the use of bandwidth and had shown to assure consistency of the kernel estimate. Section II presents the general formulation of the problem of measurement estimation. The proposed minimum-entropy scheme for parameter and measurement estimation (MEE) is described in Section III, jointly with the procedure used for uncertainty characterization. Section IV contains a brief description of the three kernel estimates compared in this work. A simple measurement example is given in Section V, and some concluding remarks about the example results are given in Section VI. A complex measurement problem of groove dimensioning using remote field Eddy current (RFEC) inspection is described in Section VII. Finally, some general conclusions are proposed in Section VIII.

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The authors are with the École Supérieure d'Électricité, Service des Mesures, Yvette Cedex, France.

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II. Problem Statement

In many applications, unknown quantities $\boldsymbol{m}$ have to be estimated from a vector of observed values $\boldsymbol{y}$. This may be encountered in several domains, such as non destructive testing or so-called indirect measurement. It is due to the inability to use transducers to measure $\boldsymbol{m}$ directly for any reason, such as harsh environments, long distance, or others. A measurement can be defined as the best way to take advantage of the information given by the observed data $\boldsymbol{y}$. The first step of a measurement procedure consists of modeling the physical phenomenon in concern. Therefore, building a model becomes a goal on its own. Measurement systems can be formalized by two equations [1].

i) The *Observation Equation* given by the classical nonlinear regression model

$$y_i = f(x_i, \boldsymbol{\theta}) + e_i, \quad i = 1, \ldots, n \tag{1}$$

where $\{x_i\}_{i=1}^n$ is an experimental design vector and $f(\cdot)$ is a known model with an unknown parameters vector $\boldsymbol{\theta}$ (p dimensions). Some fitting technique to estimate $\boldsymbol{\theta}$ can be used, for example, for nonlinear least squares (NLS), maximum likelihood (ML), M-estimator [11], etc.

ii) The *Measurement Equation* (which is a nonlinear function of $\boldsymbol{\theta}$)

$$m_\ell = g_\ell(\boldsymbol{\theta}), \quad \ell = 1, \ldots, r, \quad \text{with } \boldsymbol{m} = \{\mathrm{m}_\ell\}_{\ell=1}^{\mathrm{r}}. \tag{2}$$

The measurement is usually defined by a functional of the parametric model $m_\ell = \mathcal{G}_\ell(f)$ (i.e., derivation, integration, interpolation, extrapolation, etc.). This relation is then transformed into a function of the parameters $\boldsymbol{\theta}$ such as in (2). It is supposed that the measurement depends on at least one of the parameters $(\forall \ell, \exists k / \partial g_\ell / \partial \theta_k \neq 0)$.