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Density Estimation for Measurement Purposes and Convergence Improvement using MCMC

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Abstract* – The purpose of this paper is to present a new approach for measurement uncertainty characterization. The Markov Chain Monte Carlo (MCMC) is applied to measurement pdf estimation, which is considered as an inverse problem. The measurement characterization is driven by the pdf estimation in a non-linear Gaussian framework with unknown variance and with limited observed data. Multidimensional integration and support searching, are driven by the Metropolis-Hastings (M-H) autoregressive algorithm which performance is generally better than the M-H random walk. These techniques are applied to a realistic measurement problem of groove dimensioning using Remote Field Eddy Current (RFEC) inspection. The application of resampling methods such as bootstrap and the perfect sampling for convergence diagnostics purposes, gives large improvements in the accuracy of the MCMC estimates.

I. INTRODUCTION

In many industrial applications, direct access to a measurement (m) is not possible, this is due to the inability to use transducers to measure m directly for any reason of harsh environment, long distance or other. Thus, the measurement process must be considered as an inverse problem [1], since the measurement estimation is needed. The characterization of all statistical knowledge upon this quantity of interest is naturally driven by the probability density function (pdf) $\wp(m)$. Probabilistic inferences using Markov Chain Monte Carlo (MCMC) methods are considered as another Monte Carlo simulation technique and — *other measurement pdf estimation alternative* — by using a fully Bayesian framework [2], [3], [4]. The "Metropolis algorithm" has been used to solve difficult problems in statistical physics for over forty years (1953), a generalization of this algorithm is introduced by "Metropolis-Hastings algorithms" (1970). In the last few years, other related method of "Gibbs sampling" has been applied to problems of statistical inference. The MCMC methods, such as Gibbs sampling and Metropolis-Hastings (M-H) algorithms are powerful Markov chain methods to simulate multivariate distributions and they have a real impact on Bayesian statistics [5], [6]. The M-H algorithms have been extensively used in physics and more recently exploited by statisticians [7].

The importance of Monte Carlo methods for inference problems in signal processing has grown in recent years (see the special issue *Trans. on Signal Proc.*, vol. 50, no. 2, Feb. 2002). This is due to the explosive increase in accessible computing power. Monte Carlo methods have also a great degree of flexibility for the solution of challenging computational problems, such as optimization and integration. This kind of problems abound in statistical signal processing. Papers [8], [9] show the potentially usefulness of MCMC in signal processing. For the solution of our measurement problem, we have successfully used hybrid algorithms, firstly a Metropolis-Hastings algorithm for sampling from a complex likelihood density function, and secondly the Gibbs algorithm for sampling from the posterior density.

An important problem in MCMC is the convergence surveillance of such methods. The convergence performance of MCMC can be improved by using a resampling scheme, for example the weighted bootstrap used in [10] (see also [11], [12] that suggest a class of weighted bootstrap techniques) and perfect simulation [13] procedures (see also [14], [15]). The final interest is to apply the MCMC methods in a realistic problem of indirect measurement (measurement estimation). The section II presents the general formulation of the problem of measurement estimation and the MCMC idea extended to the measurement uncertainty characterization, the Bayesian framework for parameter and measurement estimation is described in section III, jointly with the classical MCMC uncertainty characterization. The analysis of convergence by resampling and perfect sampling methods (coupling from the past (CFTP)), are briefly described in section IV. A measurement complex problem of groove dimensioning using Remote Field Eddy Current (RFEC) inspection is given in section V, and finally some concluding remarks are given in section VI.

II. MCMC APPLIED TO MEASUREMENT PDF ESTIMATION

The problem of probability density function (pdf) estimation for an indirect measurement is considered in this work. This problem has been analyzed for a nonlinear Gaussian framework, and the results give the possibility to take up again the problem of pdf estimation in a more suitable or realistic framework (nonlinear Gaussian with unknown variance or non-gaussian). In many applications, an unknown quantity m has to be estimated from a vector of observed values $\boldsymbol{y}$. This may be encountered in several domains such as non destructive

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