Fast estimation of modulo 2π fringe orientation

Jesús Villa and Ismael de la Rosa

Laboratorio de Procesamiento Digital de Señales, Facultad de Ingeniería Eléctrica, Universidad Autónoma de Zacatecas, Av. Ramón López Velarde 801, C. P. 98000, Zacatecas, México, jvillah@uaz.edu.mx.

ABSTRACT. A regularized estimator for modulo 2π fringe orientation is presented in this work. As the technique requires to solve locally in the fringe pattern a simple linear system to optimize a regularized cost function, the global estimation of an orientation vector field is performed fast and easily. The performance of this technique is evaluated with synthetic and real fringe patterns.

1. INTRODUCTION

It is widely known that fringe pattern analysis is a simple task if a proper carrier frequency is introduced. However, in many optical tests the introduction of a carrier frequency is not possible and the automatic analysis becomes a non-trivial work. In recent years, it has been evidenced the importance of the modulo 2π fringe orientation for phase recovery [1,2], however, modulo 2π fringe orientation estimation has resulted difficult to realize [3,4].

In fringe analysis it is usual to define the image of a fringe pattern as

$$I(\mathbf{r}) = I_0(\mathbf{r}) + I_1(\mathbf{r})\cos[\phi(\mathbf{r})], \quad \mathbf{r} = (x, y) \in L, \tag{1}$$

where L is the set of valid sites in the image, $\phi(\mathbf{r})$ represents the phase to be recovered, terms $I_0(\mathbf{r})$ and $I_1(\mathbf{r})$ represent the background illumination and the varying contrast respectively. As these two terms are low frequency functions, for practical purposes they can be considered constants. Having in mind these considerations or even better eliminating these terms by means of a normalization procedure, *i. e.*, supposing $I(\mathbf{r}) \approx \cos[\phi(\mathbf{r})]$, the modulo π fringe orientation angle can be calculated with

$$\theta_\pi(\mathbf{r}) = \tan^{-1}\left[\frac{\partial I(\mathbf{r})/\partial y}{\partial I(\mathbf{r})/\partial x}\right], \qquad -\tfrac{\pi}{2} \le \theta_\pi \le \tfrac{\pi}{2}. \tag{2}$$

Due to sign ambiguity of the gradient $\nabla I(\mathbf{r})$ with respect to fringe orientation, the direct use of the modulo π fringe orientation is not proper for analyzing closed fringes.

The proposal of Larkin *et al*. [1] to demodulate closed fringes is founded in the investigation of a natural and isotropic extension to more than one dimension of the Hilbert transform. In the work of Larkin *et al*., it is presented a two-dimensional transform that involves a spiral phase spectral operator and an orientational phase spatial operator:

$$\Im^{-1}\{\exp[i\Theta(\mathbf{f})]\Im\{I(\mathbf{r})\}\} \cong i\exp[i\theta_{2\pi}(\mathbf{r})]\sin[\phi(\mathbf{r})]. \tag{3}$$

In the work reported by Servín *et al*. [2], the utility of fringe orientation is evidenced when the so-called n-dimensional quadrature operator is defined:

$$Q_n\{b(\mathbf{r})\cos[\phi(\mathbf{r})]\} = \mathbf{n}_\phi \cdot \frac{\nabla I(\mathbf{r})}{\|\nabla I\phi(\mathbf{r})\|}, \tag{4}$$

where, in the two-dimensional case

$$\mathbf{n}_{\phi} = \frac{\nabla \phi(\mathbf{r})}{\|\nabla \phi(\mathbf{r})\|} = \cos[\theta_{2\pi}(\mathbf{r})]\mathbf{i} + \sin[\theta_{2\pi}(\mathbf{r})]\mathbf{j}. \tag{5}$$

Quiroga *et al*. [4] proposed the use of the regularized phase-tracking technique for retrieving the modulo 2π fringe orientation from the modulo π fringe orientation, however it is required to solve a non-linear system in order to optimize a local cost function.

2. REGULARIZED ESTIMATOR FOR MODULO 2π FRINGE ORIENTATION

In this work we present a regularized technique to estimate a smooth vector field $\mathbf{p(r)}$ having the fringe orientation. The estimation problem presented here is formulated in the following way: find a smooth vector field $\mathbf{n(r)} = [n_x, n_y]$ orthogonal to $\boldsymbol{\psi}\mathbf{(r)} = [\cos\theta_\pi(\mathbf{r}), \sin(\theta_\pi(\mathbf{r})]$. As $\mathbf{p(r)}$ is parallel to $\boldsymbol{\psi}\mathbf{(r)}$, then $\mathbf{n(r)} \perp \mathbf{p(r)}$. Once the vector field $\mathbf{n(r)}$ is estimated, we assume that $\mathbf{p(r)} = [-n_y, n_x]$ or $\mathbf{p(r)} = [n_y, -n_x]$, and its modulo 2π orientation can be easily computed.

Estimation of $\mathbf{n(r)}$ must be carried out having in mind that

$$\boldsymbol{\psi}\mathbf{(r)} \cdot \mathbf{n(r)} = 0, \quad (\mathbf{r} \forall L). \tag{6}$$

The estimated vector field $\mathbf{n(r)}$ is regularized to restrict its smoothness in order to avoid orientation ambiguity. The regularized estimator for fringe orientation proposed here is designed considering that in a small region Γ around a coordinate site $\mathbf{r}$, the fringes point to the same direction. The proposed regularized cost function that takes into account the above-mentioned considerations is

$$U_{\mathbf{r}}(\mathbf{n}) = \sum_{\tilde{\mathbf{r}} \in (\Gamma \cap L)} \left\{ [\boldsymbol{\psi}(\tilde{\mathbf{r}}) \cdot \mathbf{n(r)}]^2 + \mu[n_x(\mathbf{r}) - n_x(\tilde{\mathbf{r}})]^2 s(\tilde{\mathbf{r}}) + \mu[n_y(\mathbf{r}) - n_y(\tilde{\mathbf{r}})]^2 s(\tilde{\mathbf{r}}) \right\}, \tag{7}$$

where $\tilde{\mathbf{r}} = (\tilde{x}, \tilde{y})$ represent the local coordinates in the region Γ, $n_x(\tilde{\mathbf{r}})$ and $n_y(\tilde{\mathbf{r}})$ represent the already computed components of the estimated vector field around site $\mathbf{r}$, $s(\tilde{\mathbf{r}})$ is an indicator that equals 1 if the site has already been estimated and 0 otherwise. The so-called regularization parameter μ together with the size of region Γ control the smoothness of the estimated vector field. To optimize the cost function (7) at every site $\mathbf{r} \in L$ we just need to solve the simple linear system

$$\nabla U_{\mathbf{r}}(\mathbf{n}) = \mathbf{0}. \tag{8}$$

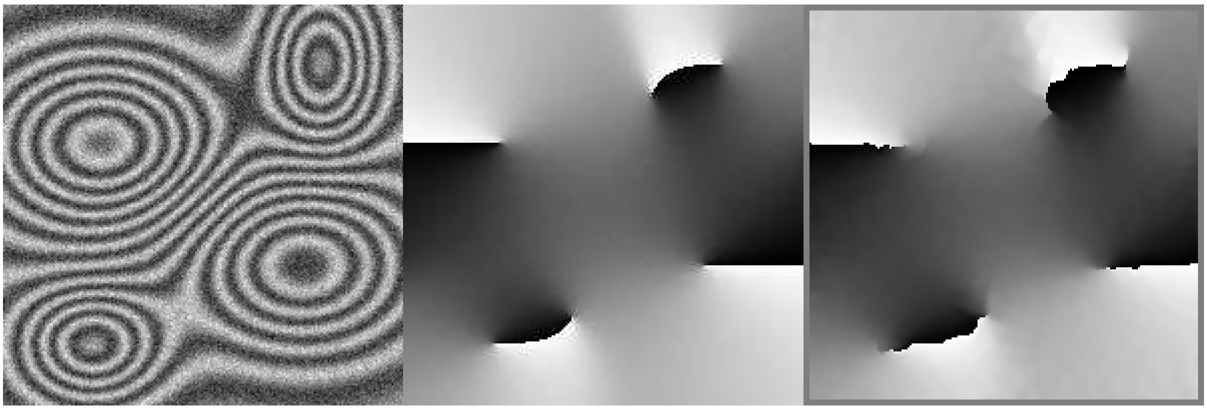
For estimating $\mathbf{n}(\mathbf{r})$ we start by setting $s(\mathbf{r}) = 0$ $(\mathbf{r} \forall L)$, and the process is carried out optimizing the cost function (7) at every site $\mathbf{r} \in L$. Once a site $\mathbf{r}$ is visited to estimate $\mathbf{n}(\mathbf{r})$, we set the indicator $s(\mathbf{r})$ equal to 1. The process is finished when all sites in L are visited. An advantageous characteristic of this estimation process is that equation (8) is a linear system, so the solution can be easily computed using a direct method.

3. EXPERIMENTS

As cost function (7) includes already estimated values of $\mathbf{n}(\mathbf{r})$, the estimation is path dependent, and better results can be obtained if smoothest or less problematic zones are processed first. This reasoning was applied by Ströbel [5] for phase unwrapping using a pixel queuing algorithm with a quality map. This pixel queuing algorithm was first adopted to be applied in the regularized phase-tracker by Villa *et al*. [6], using the amplitude modulation of squared grating deflectogram as the quality map. For our purpose, a good criterion is to follow a path that visits problematic sites of dark and bright fringe centers at last (because $\nabla \boldsymbol{I}\mathbf{(r)}$ is not well defined at this sites). Using Ströbel's pixel queuing algorithm, a proper quality map for this criterion is $\|\nabla I(\mathbf{r})\|$.

Other important detail is the first estimation in the fringe pattern. As in the first estimation there are not already estimated values of $\mathbf{n(r)} = [n_x, n_y]$, the equation (8) represent a homogeneous system, so in the experiments we propose in the first site $\mathbf{n(r)} = [-\psi_y, \psi_x]$.

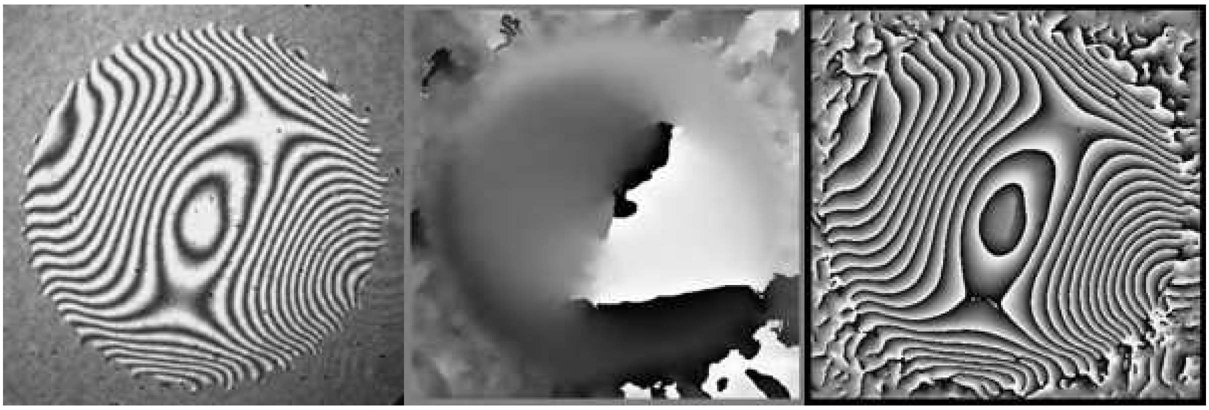
The performance of the proposed technique is illustrated by two experiments. The first is a numerical estimation with a synthetic fringe pattern of size 200×200 with 256 gray levels, shown in Figure 1(a). The image was corrupted with gaussian additive noise with a signal to noise ratio of –5.2 dB. Theoretical and estimated modulo 2π fringe orientation are shown in Figures 1(b) and 1(c) respectively. In this experiment we used the values $\mu = 1$ and $\Gamma = 7 \times 7$. The time employed for this estimation was 4 seconds using Matlab in a 3 GHz Pentium 4 based computer.



(a) (b) (b)

Figure 1. (a) Synthetic fringe pattern corrupted with gaussian additive noise. Gray level codification of (b) theoretical and (c) estimated modulo 2π fringe orientation. Black represents -π rad and white π rad.

An interferometric fringe pattern of size 450×450 is shown in Figure 2(a). The estimated modulo 2π fringe orientation is shown in Figure 2(b). The computed phase using Larkin's *et al.* operator is shown in figure 2(c). In this case we used the values $\mu = 0.5$ and $\Gamma = 13 \times 13$. The total time employed for this estimation was about 28 s.



(a) (b) (b)

Figure 2. (a) Real interferometric fringe pattern. (b) Estimated modulo 2 π fringe orientation. (c) Gray level codification of the computed phase using Larkin's *et al.* operator. Black represents - π rad and white π rad.

3. CONCLUSIONS

As shown, the technique presented here is a simple and fast manner for estimating the modulo 2π fringe orientation, and it can be applied to many kinds of fringe patterns.

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REFERENCES

[1] K. G. Larkin, D. J. Bone, and M. A. Oldfield, "Natural demodulation of two-dimensional fringe patterns. I. General background of the spiral phase quadrature transform," J. Opt. Soc. Am. A 18, 1862-1870 (2001).

[2] M. Servín, J. A. Quiroga, and J. L. Marroquín, "General n-dimensional quadrature transform and its application to interferogram demodulation," J. Opt. Soc. Am. A 20, 935-934 (2003).

[3] X. Zhou, J. P. Baird, and J. F. Arnold, "Fringe orientation estimation by use of gaussian gradient-filter and neighboring-direction averaging," Appl. Opt. 38, 795-804 (1999).

[4] J. A. Quiroga, M. Servín, and F. Cuevas, "Modulo 2π fringe orientation angle estimation by phase unwrapping with a regularized phase tracking algorithm," J. Opt. Soc. Am. A 19, 1524-1531 (2002).

[5] B. Ströbel, "Processing of interferometric phase maps as complex-valued phasor images," Appl. Opt. 35, 2192-2198 (1996).

[6] J. Villa, J. A. Quiroga, and M. Servín, "Improved regularized phase-traking technique for the processing of squared-grating deflectograms," Appl. Opt. 39, 502-508 (2000).