Steady-state sphere-like and ring-like formations of the free electromagnetic field in vacuum(*)

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Summary. — It is shown that there are exact solutions of the free Maxwell equations in vacuum allowing an existence of closed spherical magnetic surfaces (without electric field on this surface and where magnetic field is tangential and its intensity depends on time) and ring-like formations of time-dependent electric field (without magnetic field in all points of the ring and where the electric field is tangential). It is detected that a form of these spheres and rings does not change with time in vacuum. One can surmise that these electromagnetic formations correspond to Kapitsa's hypothesis about origin and a structure of *ball lightning*. It is shown a simple way to solve the equation $\nabla \times \mathbf{a} = (\omega/c)\mathbf{a}$ which is important in the theory of plasma.

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1. – Introduction

We found that a certain class of exact solutions of free Maxwell equations (FME)

$$\operatorname{div}\mathbf{E} = 0, \tag{1}$$

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

$$\operatorname{div}\mathbf{B} = 0, \tag{3}$$

$$\operatorname{rot}\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}. \tag{4}$$

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exists which has some unexpected characteristics. The present work is devoted to the research of the such solutions.

We shall look for solutions of the system FME as follows:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{e}(\mathbf{r})\psi(t) \qquad \text{and} \qquad \mathbf{B}(\mathbf{r},t) = \mathbf{b}(\mathbf{r})\chi(t), \tag{5}$$

where $\psi(t)$ and $\chi(t)$ are some functions of time, vectors $\mathbf{e}$ is a polar vector and $\mathbf{b}$ is an axial one.

As we will show in the next sections, such mathematically well-known solutions (see, *e.g.*, [1], where general solutions of the Maxwell equations was obtained) lead, however, to the existence of rather unusual and unexpected electromagnetic formations in vacuum such as closed spherical magnetic surfaces (*without* electric field on this surface and where the magnetic field is tangential and its intensity depends on time) and ring-like formations of the electric field (*without* magnetic field in all points of the ring and where the electric field is tangential and depends on time). We will also show that these formations do not change their form with time in vacuum.

2. – Solution of the free Maxwell equations in the form (5)

And so we are going to look for a solution of FME in the form (5). Substituting (5) in FME we obtain

$$\operatorname{div}\mathbf{e} = 0, \tag{6}$$

$$\operatorname{rot}\mathbf{e} = -\frac{1}{c}\frac{\chi'}{\psi}\mathbf{b}, \tag{7}$$

$$\operatorname{div}\mathbf{b} = 0, \tag{8}$$

$$\operatorname{rot}\mathbf{b} = \frac{1}{c}\frac{\psi'}{\chi}\mathbf{e}. \tag{9}$$

It is obvious that these equations are consistent if and only if

$$-\frac{\chi'}{\psi} = \omega_1 \qquad \text{and} \qquad \frac{\psi'}{\chi} = \omega_2, \tag{10}$$

where $\{'\}$ means a derivative with respect to time, ω_1 and ω_2 are arbitrary constants.

In order to obtain solutions of this system with three constants only, and to obtain sinusoidal solutions, we propose that $\omega_1 = \omega_2 = \omega$. Thus, the general solution of the system (10) is

$$\chi(t) = \mathcal{A}\cos(\omega t - \eta) \qquad \text{and} \qquad \psi(t) = \mathcal{A}\sin(\omega t - \eta), \tag{11}$$

where $\mathcal{A}$ and η are arbitrary constants, and equations for $\mathbf{e}$ and $\mathbf{b}$ become

$$\nabla\times\mathbf{e} = \frac{\omega}{c}\mathbf{b} \qquad \text{and} \qquad \nabla\times\mathbf{b} = \frac{\omega}{c}\mathbf{e}. \tag{12}$$

In order to solve this system, let us at first note that formally summing two equations (12) we obtain

$$\nabla\times(\mathbf{e}+\mathbf{b}) = \frac{\omega}{c}(\mathbf{e}+\mathbf{b}) \qquad \text{or} \qquad \nabla\times\mathbf{a} = \frac{\omega}{c}\mathbf{a}. \tag{13}$$

So, at first we resolve eq. (13) with respect to **a**, and then we obtain from the vector **a** (which, obviously, has no polarity) the polar vector **e** and the axial vector **b**. Actually, one can express polar and axial parts of any vector without polarity, in general, as follows:

$$\mathbf{e}(\mathbf{r}) = \frac{1}{2}\Big[\mathbf{a}(\mathbf{r}) - \mathbf{a}(-\mathbf{r})\Big] \tag{14}$$

and

$$\mathbf{b}(\mathbf{r}) = \frac{1}{2}\Big[\mathbf{a}(\mathbf{r}) + \mathbf{a}(-\mathbf{r})\Big]. \tag{15}$$

Now, if we calculate a rotor of both parts of eqs. (14), (15) one can be satisfied that the system (12) is fulfilled:

$$\nabla\times\mathbf{e}(\mathbf{r}) = \frac{1}{2}\Big[\nabla\times\mathbf{a}(\mathbf{r}) - \nabla\times\mathbf{a}(-\mathbf{r})\Big] = \frac{1}{2}\Big[\frac{\omega}{c}\mathbf{a}(\mathbf{r}) + \frac{\omega}{c}\mathbf{a}(-\mathbf{r})\Big] = \frac{\omega}{c}\mathbf{b}(\mathbf{r}) \tag{16}$$

and

$$\nabla\times\mathbf{b}(\mathbf{r}) = \frac{1}{2}\Big[\nabla\times\mathbf{a}(\mathbf{r}) + \nabla\times\mathbf{a}(-\mathbf{r})\Big] = \frac{1}{2}\Big[\frac{\omega}{c}\mathbf{a}(\mathbf{r}) - \frac{\omega}{c}\mathbf{a}(-\mathbf{r})\Big] = \frac{\omega}{c}\mathbf{e}(\mathbf{r}). \tag{17}$$

Here we take into account that after inverting the coordinates, the equation $\nabla\times\mathbf{a}(\mathbf{r}) = (\omega/c)\mathbf{a}(\mathbf{r})$ becomes $-\nabla\times\mathbf{a}(-\mathbf{r}) = (\omega/c)\mathbf{a}(-\mathbf{r})$. Thus, one can see that, if we find **a** as a solution of eq. (13), it means that we find **e** and **b** as solutions of the system (12).

Equation (13) was already solved in the literature (see, *e.g.*, [2,3]).

And so, the solution([1]) of eq. (13) in the spherical system of coordinates is([2])

$$\mathbf{a} = \mathcal{D}\left\{\frac{2\alpha}{r^3}\cos\theta\right\}\mathbf{e}_r + \mathcal{D}\left\{\frac{\gamma}{r^3}\sin\theta\right\}\mathbf{e}_\theta + \mathcal{D}\left\{\frac{\omega\alpha}{cr^2}\sin\theta\right\}\mathbf{e}_\varphi. \tag{18}$$

Finally, separating vectors **e** and **b** we obtain the solution of the system (12) expressed by components (Cartesian and spherical ones)

$$\mathbf{e} = \mathcal{D}\left\{-\frac{\alpha\omega y}{cr^3},\ \frac{\alpha\omega x}{cr^3},\ 0\right\} = \frac{\omega\alpha\sin\theta}{cr^2}\mathcal{D}\mathbf{e}_\varphi \tag{19}$$

and

$$\mathbf{b} = \mathcal{D}\left\{\frac{\beta xz}{r^5},\ \frac{\beta yz}{r^5},\ \frac{2\alpha}{r^3} - \frac{\beta(x^2+y^2)}{r^5}\right\} = \frac{2\alpha\cos\theta}{r^3}\mathcal{D}\mathbf{e}_r + \frac{\gamma\sin\theta}{r^3}\mathcal{D}\mathbf{e}_\theta, \tag{20}$$

where

$$\alpha = \cos\left(\frac{\omega r}{c} - \delta\right) + \frac{\omega r}{c}\sin\left(\frac{\omega r}{c} - \delta\right).$$

([1]) Details of solving of eq. (13) see in the appendix.
([2]) $\mathcal{D}$ is a dimension constant $[\mathcal{D}] = \mathtt{M}^{1/2}\mathtt{L}^{5/2}\mathtt{T}^{-1}$.

$$\beta = 3\alpha - \frac{\omega^2 r^2}{c^2}\cos\left(\frac{\omega r}{c} - \delta\right) \qquad \text{and} \qquad \gamma = \beta - 2\alpha.$$

Let us now write the solution (5) in the explicit form, taking into account eqs. (11), (19) and (20):

(21) $$\mathbf{E} = \left[\frac{\omega\alpha\sin\theta}{cr^2}\mathcal{D}\mathbf{e}_\varphi\right]\sin(\omega t - \eta)$$

and

(22) $$\mathbf{B} = \left[\frac{2\alpha\cos\theta}{r^3}\mathcal{D}\mathbf{e}_r + \frac{\gamma\sin\theta}{r^3}\mathcal{D}\mathbf{e}_\theta\right]\cos(\omega t - \eta),$$

where η is an arbitrary constant.

It follows from the solutions (21), (22) that the necessary (not sufficient!) condition in order for these solutions to not diverge in $r = 0$ is

$$\alpha(0) = \left\{\cos\left(\frac{\omega r}{c} - \delta\right) + \frac{\omega r}{c}\sin\left(\frac{\omega r}{c} - \delta\right)\right\}\Big|_{r=0} = 0 \qquad \Longrightarrow$$

$$\cos\delta = 0 \qquad \Longrightarrow \qquad \delta = (n + \frac{1}{2})\pi, \quad \text{where } n = 0, \pm 1, \pm 2, \dots .$$

In order to make sure that the solutions (21), (22) converge, one can calculate the following limits([3]) for $\delta = \pi/2$:

(23) $$\lim_{r\to 0}\frac{\alpha}{r^2} = 0; \qquad \lim_{r\to 0}\frac{\alpha}{r^3} = \frac{\omega^3}{3c^3}; \qquad \lim_{r\to 0}\frac{\gamma}{r^3} = -\frac{2\omega^3}{3c^3},$$

and the corresponding limits for $\mathbf{E}$, $\mathbf{B}$ and the energy density $w = \frac{E^2+B^2}{8\pi}$ are

(24) $$\lim_{r\to 0}\mathbf{E} = 0; \qquad \lim_{r\to 0}\mathbf{B} = \frac{2\mathcal{D}\omega^3\cos(\omega t)}{3c^3}\mathbf{k}; \qquad \lim_{r\to 0} w = \frac{\mathcal{D}^2\omega^6}{18\pi c^6}\cos^2(\omega t),$$

where $\mathbf{k}$ is *Z-ort* of the Cartesian system.

The constant η just defines an initial wave phase of the fields $\mathbf{E}$ and $\mathbf{B}$. So without loss of generality we can write one non-divergent solutions only (for $\delta = \pi/2$, $\eta = 0$):

(25) $$\mathbf{E} = \mathcal{D}\left[\frac{\alpha\omega\sin\theta}{cr^2}\mathbf{e}_\varphi\right]\sin(\omega t); \quad \mathbf{B} = \mathcal{D}\left[\frac{2\alpha\cos\theta}{r^3}\mathbf{e}_r + \frac{\gamma\sin\theta}{r^3}\mathbf{e}_\theta\right]\cos(\omega t),$$

where

(26) $$\alpha = -\frac{\omega r}{c}\cos\left(\frac{\omega r}{c}\right) + \sin\left(\frac{\omega r}{c}\right) \qquad \text{and} \qquad \gamma = \alpha - \frac{\omega^2 r^2}{c^2}\sin\left(\frac{\omega r}{c}\right).$$

Note that the solution (25) can be found directly from the general solution of the Maxwell equations obtained by Mie [1].

([3]) We calculate these limits expanding α and γ in series of powers of r.

3. – Steady-state electromagnetic balls in vacuum as a consequence of the found solution

As we will show below, the solution (25) of FME leads to an existence of unusual spherical formations of the free electromagnetic field.

3˙1. *Some details of the energy distribution in the field* (25). – Let us write, after some transformations, the expression for the energy density for the solution (25). One can show that the energy density contains both the time-independent part and the time-dependent one:

$$(27)\quad w = \frac{E^2+B^2}{8\pi} = \frac{\mathcal{D}^2}{16\pi}\left\{\frac{\omega^2\alpha^2}{c^2r^4}\sin^2\theta + \left[\frac{4\alpha^2}{r^6}\cos^2\theta + \frac{\gamma^2}{r^6}\sin^2\theta\right]\right\} + \\ + \frac{\mathcal{D}^2}{16\pi}\left\{\left[\frac{4\alpha^2}{r^6}\cos^2\theta + \frac{\gamma^2}{r^6}\sin^2\theta\right] - \frac{\omega^2\alpha^2}{c^2r^4}\sin^2\theta\right\}\cos(2\omega t).$$

Let us find from (27) the *locus* where w does not depend on t. It is obvious that the *loci* are

1) along the axis Z in the points where $\tan\left(\frac{\omega z}{c}\right) = \frac{\omega z}{c}$ ($\theta = 0, \pi; \alpha = 0$);

2) at surfaces where r satisfies the equation $\gamma^2 = \alpha^2\left(\frac{\omega^2 r^2}{c^2} - 4\cot^2\theta\right)$. One can see the cross-section of these surfaces in fig. 3 (discontinuous curves)([4]).

Now we calculate an electromagnetic energy $\mathcal{E}$ within a sphere of the radius R with the center in the coordinate origin:

$$(28)\quad \mathcal{E}_{\oplus} = \int_0^R \mathrm{d}r \int_0^\pi \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\varphi\, r^2 \sin\theta\, w(r,\theta,\varphi,t) = \mathcal{E}(R) + \mathcal{E}(R,t),$$

where

$$(29)\quad \mathcal{E}(R) = \frac{\mathcal{D}^2}{6R^3}\left[\frac{\omega^4 R^4}{c^4} - \frac{\omega^2 R^2}{c^2}\sin^2\left(\frac{\omega R}{c}\right) - \alpha^2\right],$$

$$(30)\quad \mathcal{E}(R,t) = -\frac{\mathcal{D}^2}{6R^3}\alpha\gamma\cos(2\omega t).$$

Here $\alpha = -\frac{\omega R}{c}\cos\left(\frac{\omega R}{c}\right) + \sin\left(\frac{\omega R}{c}\right)$ and $\gamma = \alpha - \frac{\omega^2 R^2}{c^2}\cos\left(\frac{\omega R}{c}\right)$.

One can show from eq. (30) that the electromagnetic energy within spheres of the radiuses R which are solutions of the equations([5])

$$(31)\quad \tan\left(\frac{\omega R}{c}\right) = \frac{\omega R}{c},$$

([4]) All figures in this work were performed in the program "Mathematica-4.0".
([5]) It follows from $\alpha = 0$ and $\gamma = 0$ correspondingly.

or

$$\tan\left(\frac{\omega R}{c}\right) = \frac{\frac{\omega R}{c}}{1 - \frac{\omega^2 R^2}{c^2}}, \tag{32}$$

does not change with time.

The solutions of eqs. (31), (32) alternate with each other at the number line. One can show that a distance between these neighboring spherical surfaces tends to $c\pi/2\omega$ when $R \to \infty$. Let us also direct attention to an interesting fact that at the surfaces of the spheres of the radius (31) only the magnetic field is present, and the electric field at these surfaces does not exist. It follows directly from eq. (25) for $\alpha = 0$.

3·2. *Analysis of the Poynting vector's field corresponding to the wave field* (25). – Poynting's vector corresponding to the wave field (25) is

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B} = \frac{\mathcal{D}^2}{8\pi}\left[\frac{\omega\alpha^2\sin(2\theta)}{r^5}\mathbf{e}_\theta - \frac{\omega\alpha\gamma\sin^2\theta}{r^5}\mathbf{e}_r\right]\sin(2\omega t). \tag{33}$$

Let us calculate a total momentum of the electromagnetic field (25) within a sphere of the arbitrary radius r with the center in the coordinate origin. Because Poynting's vector is proportional to the vector of the density of momentum in the same point we can just calculate the integral of Poynting's vector over volume of the sphere.

It is easy to calculate this integral if we express spherical system *orts* by Cartesian system *orts*:

$$\mathbf{e}_r = \mathbf{i}\sin\theta\cos\varphi + \mathbf{j}\sin\theta\sin\varphi + \mathbf{k}\cos\theta \quad \text{and} \quad \mathbf{e}_\theta = \mathbf{i}\cos\theta\cos\varphi + \mathbf{j}\cos\theta\sin\varphi - \mathbf{k}\sin\theta.$$

Thus, integrating (33) over volume of the sphere we obtain

$$\iiint \mathbf{S}\, r^2\sin\theta\, \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}\varphi = -\frac{\mathcal{D}^2 4\pi\omega\sin(2\omega t)}{32\pi}\mathbf{k}\int \frac{\alpha^2}{r^3}\sin^4\theta\Big|_0^\pi \mathrm{d}r = 0. \tag{34}$$

It means that the total momentum of the electromagnetic field (25) in volume bounded by an arbitrary sphere with a center in the coordinate origin *is zero* at any time.

Let us now find the *loci* where Poynting's vector is zero at any instant of time. It follows from eq. (33) that conditions when Poynting's vector is zero are

$$\alpha^2\sin(2\theta) = 0 \qquad \text{and} \qquad \alpha\gamma\sin^2\theta = 0. \tag{35}$$

From the first equation of the conditions (35) we have the following possibilities:

i) $\alpha = 0$. This automatically satisfies both conditions (35). From $\alpha = 0$ we obtain the equation

$$\tan\left(\frac{\omega r}{c}\right) = \frac{\omega r}{c}. \tag{36}$$

Hence, the *loci* for the case i) are spheres whose radiuses satisfy eq.(36).

ii) $\sin(2\theta) = 0$. It means that θ can be 0, $\pi/2$ or π.

ii-1) If θ is 0 or π, in this case both equations fulfill the conditions (35). So the *locus* is axis Z.

ii-2) If $\theta = \pi/2$, this gives us two possibilities in order to satisfy the conditions (35): *either* $\alpha = 0$ (it is the case i), see above) *or* $\gamma = 0$. From the last we have

$$\tan\left(\frac{\omega r}{c}\right) = \frac{\frac{\omega r}{c}}{1 - \frac{\omega^2 r^2}{c^2}}. \tag{37}$$

So the *loci* corresponding the case $\theta = \pi/2$, and $\gamma = 0$ are rings at the plane XY with radiuses satisfying eq. (37). Note that in all points of these *rings* the magnetic field is *zero*.

Now we consider spheres whose equators are mentioned *rings*. These spheres are defined by the condition $\gamma = 0$. One can see from eq. (33) that at these surfaces Poynting's vector in all points has the tangential components only. Due to this fact the conservation of the energy within spheres of the radiuses (32) becomes more clear.

Thus, the adjusted total in looking for the *loci* where Poynting's vector for the field (25) is zero in any instant of time is

Locus 1: Axis Z. We call this axis *magnetic axis* because an electric field does not exist there.

Locus 2: Rings at the plane $z = 0$ with radiuses satisfying eq. (37). We call these rings *electric rings* because a magnetic field does not exist there.

Locus 3: Spheres with centers in the origin with radiuses satisfying eq. (36). We call these spheres *magnetic spheres* because the an electric field does not exist on them.

In order to elucidate better the results of the last analysis, let us adduce the graphic (fig. 1) where the distribution of Poynting's vector field is shown.

We consider this distribution, for example, in the plane $x = 0$ (because of the axial symmetry of the energy density and energy-flux density distribution it is sufficient to consider this cross-section only).

We call spheres whose equator is the *electric ring* E-sphere. We call the *magnetic spheres* M-spheres. In fig. 1 one can see the vertical *magnetic axis* (coinciding with the Z-axis), the first E-sphere, the first M-sphere and the second E-sphere in the given instant of time. Within E-spheres the total electromagnetic energy conserves because the energy-flux vector at the surface of this sphere has tangential component only. The energy transfers along this surface from pole to equator (*electric ring*) and after a certain period([6]) of time does reverse movement. Within the first E-sphere the energy transfers from the *magnetic axis* to the *electric ring* and after a certain time returns.

The Poynting vector is zero in every point of the first M-sphere so the energy within this sphere conserves too. One can see that the energy transfers from the surface of the first *magnetic sphere* to the *electric rings* of the first and the second E-spheres. Analogical exchange of the energy takes place between next E- and M-spheres.

We once more emphasize that Poynting's vector field takes opposite direction with time, due to the existence of the function $\sin(2\omega t)$ in eq. (33).

For more demonstrativeness we adduce here the graphic (fig. 2) of cross-section of Poynting's vector field in the plane $z = 0$.

([6]) This period is defined by the function $\sin(2\omega t)$ from eq. (33).

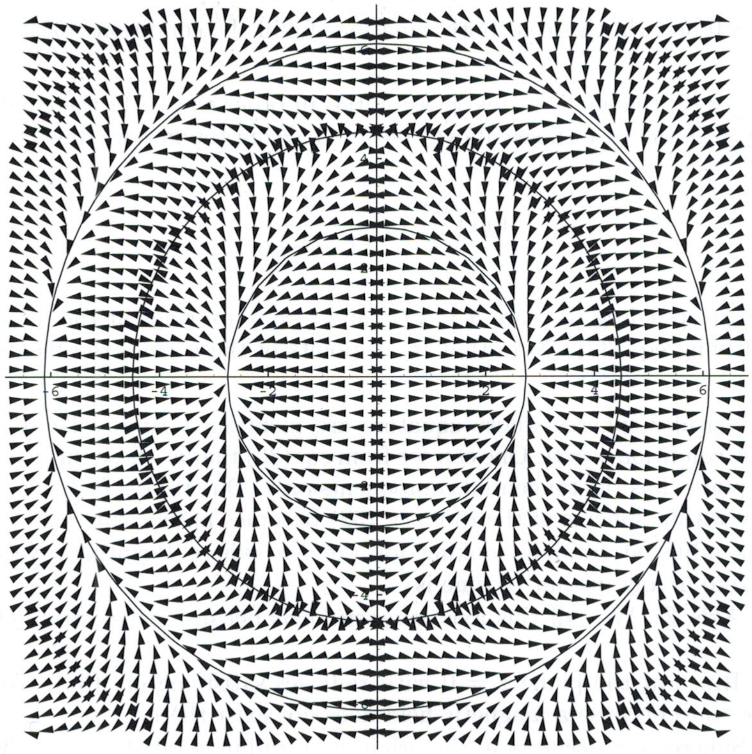

Fig. 1. – Poynting's vector field distribution for given instant of time in the plane $x = 0$, Y-axis is the abscissa and Z-axis is the ordinate. Here $c = 1, \omega = 1$.

At last we adduce here the common graphic (fig. 3) of cross-section ($x = 0$) of the surfaces where the energy density is constant and first M-sphere, first and second E-spheres.

We emphasize that these surfaces do not deform, do not displace and do not rotate with time in vacuum.

4. – Discussion

Thus, we obtained a stationary-*free* electromagnetic field which can be consequence of some interference processes. Why one can speak here about interference? Actually, we see that in this electromagnetic formation, surfaces (discontinuous curves in fig. 3) and points (in Z-axis) where the energy density is *constant* exist. From this one can surmise that these surfaces and points are nodes of wave. It is well known also that standing electromagnetic waves are a result of interference processes.

Of course, the solution of the *free* Maxwell equations corresponding to these ball-like electromagnetic formations was obtained for vacuum. But if we call to mind that in air, the values $\varepsilon = 1$, $\mu = 1$ we can be practically sure that the solution (25) is valid for air, taking into account that air does not have free charges and currents. So it is easy to draw an analogy between our solution and Kapitsa's hypothesis about the *interference nature* of ball lightning [4]. Actually, the electric field of electromagnetic waves which "voyage"

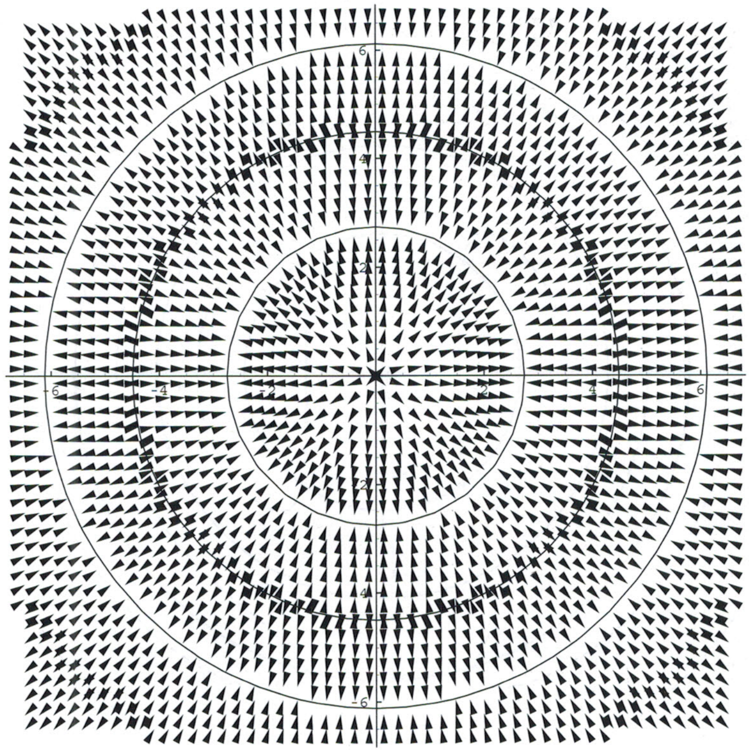

Fig. 2. – Poynting's vector field distribution for a given instant of time in the plane $z = 0$, X-axis is the abscissa and Y-axis is the ordinate.

within M-spheres and especially the electric field of the aforementioned *electric rings* have to ionize the air converting it to plasma. A size of the critical region of ionization is defined by the radius of the *magnetic sphere*, in which a density energy is still adequate for ionizing air. This ultimate *magnetic sphere* in turn plays the role of a magnetic trap for plasma confinement. One can indeed see from eq. (27) that the energy density within the magnetic spheres decreases as $1/r^2$. It means that at a certain distance the energy density is less than the critical value which is necessary to ionize the air. This condition has to define the radius of the ultimate magnetic sphere within which conditions of the ionization still exist. Taking into account this limited value of the radius of this ultimate magnetic sphere one can speak about the fireballs.

It goes without saying that it is just our hypothesis, but the analogy between Kapitsa's idea and our *ball-like* solutions doubtless takes place. It should also be stated that other ball-like stable formations in the radiation field were obtained in the paper "Is there yet an explanation of ball lightning?" by Arnhoff [5] and in the paper "Ball lightning as a force-free magnetic knot" by Rañada *et al.* [6] (see also [7]). It follows from these works that the electromagnetic energy contained in a spherical volume, cannot escape (the energy corresponding to our solutions behaves in the same way). According to [5] outside of this volume there is only a quasi-electrostatic field, rotating with constant angular velocity about the axis (in this point our and Arnhoff's solutions are different). In turn Rañada *et al.* [6, 7] proposed ball-like electromagnetic formations as a solution

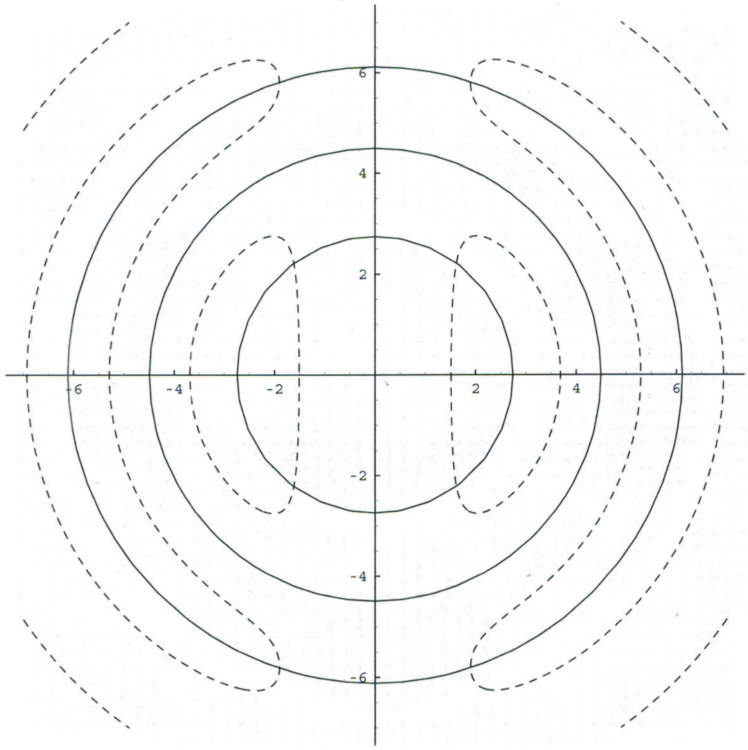

Fig. 3. – Cross-section of the surfaces of the constant energy density and first M-sphere and first E-spheres in the plane $x = 0$, Y-axis is the abscissa and Z-axis is the ordinate. Here $c = 1, \omega = 1$.

based on the idea of the "electromagnetic knot", an electromagnetic field in which any pair of magnetic lines or any pair of electric lines form a link—a pair of linked curves.

Thus the famous hypothesis of the Nobel prizewinner Kapitsa that fireballs (or balls lightning) are standing electromagnetic waves of unusual configuration as a result of some *interference* process from the day of its formulation (in 1955) never (to the present day) got a theoretical (mathematical) support. One can see that our work first gives a theoretical support to this hypothesis.

In a subsequent work we are going to research the process of the genesis of these unusual electromagnetic formations.

And in conclusion we just note that Barut was right when claimed that "*Electrodynamics and the classical theory of fields remain very much alive and continue to be the source of inspiration for much of the modern research work in new physical theories*" [8].

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Appendix

Simple solving of the equation $\nabla \times \mathbf{a} = (\omega/c)\mathbf{a}$

In spite of the fact that this equation was already solved in the literature (see, *e.g.*, [2, 3]) we decide to adduce here a different and very simple method of the solution of this vector equation.

One can make sure that a simple way to obtain a solution of eq. (13) exists, if we represent the vector **a** in the spherical system of coordinates as an axial-symmetric vector:

$$\mathbf{a} = a_r(r,\theta)\mathbf{e}_r + a_\theta(r,\theta)\mathbf{e}_\theta + a_\varphi(r,\theta)\mathbf{e}_\varphi. \tag{A.1}$$

The rotor of a vector in spherical coordinates out of the origin is

$$\nabla \times \mathbf{a} = \frac{\mathbf{e}_r}{r^2 \sin\theta}\left[\frac{\partial(ra_\varphi \sin\theta)}{\partial\theta} - \frac{\partial(ra_\theta)}{\partial\varphi}\right] + \frac{\mathbf{e}_\theta}{r\sin\theta}\left[\frac{\partial(a_r)}{\partial\varphi} - \frac{\partial(ra_\varphi \sin\theta)}{\partial r}\right] + \\ + \frac{\mathbf{e}_\varphi}{r}\left[\frac{\partial(ra_\theta)}{\partial r} - \frac{\partial a_r}{\partial\theta}\right]. \tag{A.2}$$

Taking into account eq. (13) and comparing (A.1) and (A.2) we obtain the following system:

$$\begin{cases} \dfrac{\partial(a_\varphi \sin\theta)}{\partial\theta} = \dfrac{\omega r a_r \sin\theta}{c}, \\ \dfrac{\partial(ra_\varphi)}{\partial r} = -\dfrac{\omega r a_\theta}{c}, \\ \dfrac{\partial(ra_\theta)}{\partial r} - \dfrac{\partial a_r}{\partial\theta} = \dfrac{\omega r a_\varphi}{c}. \end{cases} \tag{A.3}$$

From the system (A.3) one can obtain a differential equation for a_φ only:

$$r\frac{\partial^2}{\partial r^2}(ra_\varphi) + \frac{\partial}{\partial\theta}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(a_\varphi \sin\theta)\right] + \frac{\omega^2 r^2}{c^2}a_\varphi = 0. \tag{A.4}$$

If we look for the solution of eq. (A.4) in the form

$$a_\varphi = R(r)\Theta(\theta), \tag{A.5}$$

we obtain that these functions have to satisfy the following equations, respectively:

$$r^2\frac{\mathrm{d}^2(rR)}{\mathrm{d}r^2} + \left(\frac{\omega^2 r^2}{c^2} + p\right) rR = 0 \tag{A.6}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}(\Theta \sin\theta)\right] - p\Theta = 0, \tag{A.7}$$

where p is an arbitrary constant. If p were zero, the solution for rR in eq. (A.6) would be $\mathcal{A}\cos\frac{\omega r}{c}+\mathcal{B}\sin\frac{\omega r}{c}$ ($\mathcal{A}$ and $\mathcal{B}$ are constants). Accordingly, in general, we are going to look for the solution of eq. (A.6) in the form

$$rR = A(r)\cos\frac{\omega r}{c} + B(r)\sin\frac{\omega r}{c}; \tag{A.8}$$

here $A(r)$ and $B(r)$ are some functions of r. Substituting (A.8) in eq. (A.6) and taking into account that coefficients of sine and cosine (which have the same argument) must be equal to *zero* separately, we obtain system of two ordinary differential equations:

$$A'' + \frac{p}{r^2}A + \frac{2\omega}{c}B' = 0 \qquad \text{and} \qquad B'' + \frac{p}{r^2}B - \frac{2\omega}{c}A' = 0. \tag{A.9}$$

Let us propose that $A(r) = \mu r^m$ and $B(r) = \nu r^n$, where μ, ν, m, n are constants and m, n are integer. Substituting these values in eqs. (A.9) we obtain "characteristic" equations:

$$\mu m(m-1) + p\mu + \frac{2\omega}{c}\nu n r^{n-m+1} = 0, \quad \nu n(n-1) + p\nu - \frac{2\omega}{c}\mu m r^{m-n+1} = 0, \tag{A.10}$$

that one can verify in two following cases only:

I) $m = 0, n = -1, p = -2, \mu = -(\omega/c)\nu$, and taking into account eq. (A.8) we obtain for $\nu = 1$

$$R = \frac{1}{r^2}\left(-\frac{\omega r}{c}\cos\frac{\omega r}{c} + \sin\frac{\omega r}{c}\right); \tag{A.11}$$

II) $m = -1, n = 0, p = -2, \nu = (\omega/c)\mu$, and taking into account eq. (A.8) we obtain for $\mu = 1$

$$R = \frac{1}{r^2}\left(\cos\frac{\omega r}{c} + \frac{\omega r}{c}\sin\frac{\omega r}{c}\right). \tag{A.12}$$

So the general soluion of eq. (A.6) for Rr is

$$R(r) = \frac{\mathcal{C}_1}{r^2}\left(-\frac{\omega r}{c}\cos\frac{\omega r}{c} + \sin\frac{\omega r}{c}\right) + \frac{\mathcal{C}_2}{r^2}\left(\cos\frac{\omega r}{c} + \frac{\omega r}{c}\sin\frac{\omega r}{c}\right), \tag{A.13}$$

where $\mathcal{C}_1$ and $\mathcal{C}_2$ are arbitrary constants. The solution (A.13) can be expressed in the form

$$R(r) = \frac{\mathcal{C}}{r^2}\left[\cos\left(\frac{\omega r}{c} - \delta\right) + \frac{\omega r}{c}\sin\left(\frac{\omega r}{c} - \delta\right)\right], \tag{A.14}$$

where $\mathcal{C}$ and δ are arbitrary constants.

Now eq. (A.7) becomes ($p = -2$)

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left[\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}(\Theta\sin\theta)\right] + 2\Theta = 0. \tag{A.15}$$

Its general solution is

$$\Theta(\theta) = \mathcal{C}_3\sin\theta + \mathcal{C}_4\big(\cot\theta - \sin\theta\ \ln|\csc\theta - \cot\theta|\big). \tag{A.16}$$

As a particular case we take the values $\mathcal{C}_4 = 0$ (because a corresponding solution has a singularity in $\theta = (2n+1)\pi$) and $\mathcal{C}_3 = 1$ (by virtue of homogeneity of the equation for the vector $\mathbf{a}$).

Thus, we can write the solution (A.5) as follows:

$$a_\varphi(r,\theta) = \frac{\alpha}{r^2}\sin\theta, \tag{A.17}$$

where

$$\alpha = \cos\left(\frac{\omega r}{c} - \delta\right) + \frac{\omega r}{c}\sin\left(\frac{\omega r}{c} - \delta\right).$$

Now, using eqs. (A.3), we can find $a_r(r,\theta)$ and $a_\theta(r,\theta)$:

$$a_r(r,\theta) = \frac{2c\alpha}{\omega r^3}\cos\theta, \qquad a_\theta(r,\theta) = \frac{c\gamma}{\omega r^3}\sin\theta, \tag{A.18}$$

where

$$\gamma = \alpha - \frac{\omega^2 r^2}{c^2}\cos\left(\frac{\omega r}{c} - \delta\right).$$

And so, we have found the solution of eq. (13) which in the spherical system of coordinates is([7])

$$\mathbf{a} = \mathcal{D}\left\{\frac{2\alpha}{r^3}\cos\theta\right\}\mathbf{e}_r + \mathcal{D}\left\{\frac{\gamma}{r^3}\sin\theta\right\}\mathbf{e}_\theta + \mathcal{D}\left\{\frac{\omega\alpha}{cr^2}\sin\theta\right\}\mathbf{e}_\varphi. \tag{A.19}$$

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([7]) For convenience we multiplied the solution by $\mathcal{D}\frac{\omega}{c}$.