Classical Electrodynamics: The Problems in the Theoretical Description of the Intra-Dipole Radiation

Abstract

In our paper we would like to analyze some mathematical and physical problems which arise in the interior of an electric dipole during its oscillation along the vector of the dipole moment. A hypothesis is advanced that only electric dipole can radiate electromagnetic waves rather than an electric charge.

Keywords: Classical electrodynamics; Intra-Dipole radiation; Velocity; Acceleration; Poynting vector; Electromagnetic field; Density; Umov vector; Dipoles; Charges; Mathematical physics; Lorentz condition; D'alembert's equation; Homogeneous wave equations

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Introduction

The present paper was inspired by our careful perusal of recently published brilliant work [1]. In our paper we are going to try to investigate what happens to the transfer of energy and momentum from one to another dipole charge during longitudinal oscillations of one of the charges. Let the dipole $\{+q,-q\}$ lies on the *X*-axis. Let also one of the charges is oscillating in some arbitrary way along the *X*-axis.

An electric field created by an arbitrarily moving charge is given by the following expression obtained directly from Lienard-Wiechert potentials [2]:

$$\mathbf{E}(\mathbf{R},t) = q\frac{\left(\mathbf{R}-R\frac{\mathbf{v}}{c}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(R-\mathbf{R}\frac{\mathbf{v}}{c}\right)^3} + q\frac{\left\{\mathbf{R}\times\left[\left(\mathbf{R}-R\frac{\mathbf{v}}{c}\right)\times\frac{\dot{\mathbf{v}}}{c^2}\right]\right\}}{\left(R-\mathbf{R}\frac{\mathbf{v}}{c}\right)^3}, \qquad (1)$$

Where $\mathbf{R}$ the vector is directed from the charge q to the point of observation, $\mathbf{v}$ and $\dot{\mathbf{v}}$ are the velocity and the acceleration of the charge q, respectively. All values in the right-hand are taken in the moment of time $t_0 = t-\tau$, where τ the retarded time is, and t is time of observation. Since along the *X*-axis all vectors in (1) are collinear, the second term in (1) is zero. In the conventional theory, the Poynting vector represents electromagnetic field energy flow per unit area per unit time across a given surface,

$$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E}\times\mathbf{H}), \quad \mathbf{p} = \frac{1}{c^2}\mathbf{S}, \qquad (2)$$

Where $\mathbf{S}$ is the Poynting vector, $\mathbf{p}$ is the momentum density vector, $\mathbf{E}$ and $\mathbf{H}$ are strengths of electric and magnetic field, respectively. Analyzing (2), one can easily note that $\mathbf{S}$ and $\mathbf{p}$ (and, therefore, all electromagnetic energy flow) are exactly zero $(\mathbf{S}=0)$ along the *X*-axis. On the other hand, from the energy conservation law,

$$w = \frac{E^2+H^2}{8\pi}, \qquad \frac{\partial w}{\partial t} = -\nabla\ .\ \mathbf{S}, \qquad (3)$$

Where w is the energy density of the electromagnetic field $\mathbf{E}$ and $\mathbf{H}$, we conclude that w and $\partial w/\partial t$ should differ from zero everywhere along the *X*-axis because there is a linear relationship between w and E^2 changing in time along the *X*-axis. An ambiguity takes place if any dipole charge is moving in some arbitrary way along the *X*-axis. As a result the energy density w should also alter as a function of changing electric field E. Then the question logically arises: *what is the mechanism that changes electric field at some fixed distance from the charge on the X-axis if there is apparently no electromagnetic field energy transfer in that direction* $(\mathbf{S}=0)$? This ambiguity is due to the fact that in the conventional theory based on the of local field, which energy has to be stored locally in space, any change of field components is indispensable without field energy flux. This is obviously violated in the above mentioned example that brings into question an assumed sufficiency of transverse solutions alone to describe all properties of electromagnetic field. At least, the resolution of this ambiguity cannot be based on transverse solutions of Maxwell's equations because it well-established that any moving charge does not radiate electromagnetic waves along the direction of its motion. Only longitudinal components, if they exist, can be useful in that respect.

Let us make several qualitative observations on the possible role of longitudinal fields components. The solution (1) indicates the existence of longitudinal perturbations along the *X*-axis. It is believed that the energy transfer (the Poynting vector) $\mathbf{S}$ is a product of the energy density and its spreading velocity $c\mathbf{n}$, (the Umov vector $\mathbf{U}$).

$$\mathbf{S} = \mathbf{U} = wc\mathbf{n}, \qquad (4)$$

Where c is the velocity of light, and $\mathbf{n}$ is the unit vector in the direction of spreading of the energy, then either the spreading velocity $c\mathbf{n}$ or the energy density w must would be zero along the *X*-axis. The first assumption would neglect any possibility of interaction transfer. The second one $(w = 0)$ would be inconceivable in the framework of Faraday-Maxwell local field which should be locally stored in space with non-zero energy. But we adduce here the theorem "*For the equality of the Poynting vector and Umov vector it is necessary and sufficient that* $\mathbf{E} \perp \mathbf{H}$ and $E = H$ " which was proved by one of the authors (Augusto Espinoza) of the present paper in [3]. The proof is very simple:

Let us study what condition in vacuum for $\mathbf{E}$ and $\mathbf{H}$ in an electromagnetic wave must be satisfied when the equality $\mathbf{S} = \mathbf{U}$ is valid. We have in CGS (Gauss' system):

$$\mathbf{S} = \frac{c}{4\pi}(\mathbf{E}\times\mathbf{H}) = \frac{c}{4\pi}EH \sin\alpha\mathbf{n} \qquad (5)$$

and

$$\mathbf{U} = wc\mathbf{n} = \frac{c}{4\pi}\left(E^2+H^2\right)\mathbf{n}, \qquad (6)$$

Here $\mathbf{n}$ is a unit vector along the direction of spreading of the electromagnetic energy, the transferring energy velocity in the case of electromagnetic waves in vacuum is c .

Equating (5) and (6) we obtain

$$2EH \sin\alpha = E^2 + H^2 \qquad (7)$$

Or

$$(E-H)^2 + 2EH(1-\sin\alpha) = 0. \qquad (8)$$

According to the problem definition we choose real values of E, H and α only, where α is the angle between $\mathbf{E}$ and $\mathbf{H}$. Therefore, the last equality (8) can be valid if and only if $E = H$ and $\alpha = \pi/2$. In this work [3] it is described the experiment performed by the authors of [3]. So for the case examined by us there is an incompatibility between the generally accepted definition of the electromagnetic energy density and the conventional definition of the energy flux density expressed by the Poynting vector. This particular case allows us to affirm that, *in general*, these standard definitions for $\mathbf{S}$ and for $\mathbf{U}$ are *incompatible*.

So we must conclude that in our case for the transfer of energy along the *X*-axis from the the oscillating charge ($+q$, for example) of the dipole $\{+q,-q\}$ to the second charge $(-q)$ the Poynting vector is not responsible, but only Umov vector.

At the end of this Section, we stress that in the conventional electrodynamics longitudinal field components in vacuum do not play any role at all and, in fact, they are eliminated from consideration by means of appropriate gauge. In Dirac's own words [4]: "*...As long as we are dealing only with transverse waves, we cannot bring in the Coulomb interactions between particles. To bring them in, we have to introduce longitudinal electromagnetic waves: The longitudinal waves can be eliminated by means of mathematical transformation. Now, when we do make this transformation which results in eliminating the longitudinal electromagnetic waves, we get a new term appearing in the Hamiltonian. This new term is just the Coulomb energy of interaction between all the charged particles,*

$$\sum_{1,2}\frac{q_1 q_2}{\mathbf{r}_{1,2}}, \qquad (9)$$

... this term appears automatically when we make the transformation of the elimination of the longitudinal waves." As we know from the classical physics, (9) means the existence of bipartite instantaneous longitudinal interaction with no potential energy stored locally in the interparticle space. What is then the meaning of the elimination of longitudinal components in the conventional theory? In the following we will try to show that the problem of longitudinal components is unreasonably underestimated in classical electrodynamics (perhaps by historical reasons). There should be a change of attitude towards its status. Mathematical and physical reasons in favor of paramount importance of longitudinal components to build up a self-consistent classical electrodynamics and its possible reconciliation with quantum mechanics will be given in next sections.

Mathematical foundations of electrodynamics with longitudinal interactions

Let us recall that a complete set of Maxwell's equations in vacuum is

$$\nabla\cdot\mathbf{E} = 4\pi\tilde{n}, \qquad (10)$$

$$\nabla\cdot\mathbf{H} = 0, \qquad (11)$$

$$\nabla\times\mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \qquad (12)$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}. \qquad (13)$$

If this system of equations is really complete and boundary conditions are adequate, it should describe all electromagnetic phenomena without exceptions and ambiguities. It is often convenient to introduce potentials, satisfying the Lorentz condition

$$\nabla\cdot\mathbf{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0. \qquad (14)$$

As a result, the set of coupled first-order partial differential equations (10)-(13) can be reduced to the equivalent pair of uncoupled inhomogeneous D'Alembert's equations:

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -4\pi\tilde{n}(\mathbf{r},t), \qquad (15)$$

$$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\mathbf{j}(\mathbf{r},t). \qquad (16)$$

Differential equations have, generally speaking, an infinite number of possible solutions. A uniquely determined solution is selected by laying down sufficient additional conditions. Different forms of additional conditions are possible for the second order partial differential equations: initial value and boundary-value conditions. A general solution of the D'Alembert equation is considered as an explicit time-dependent function of the type $\mathbf{g}(\mathbf{R},t)$. Let us discuss a very subtle point related to the use and interpretation of *implicit* and *explicit* time dependencies in the conventional electrodynamics. We think that as far as this problem is not cleared up, the classical theory will remain beset of ambiguities. Helmholtz-type approach [5] (see also the paper "The Contribution of Hermann von Helmholtz to Electrodynamics" [6]) reviewed below makes that distinction very clear.

Special relativity well established that in the stationary approximation (charge moving with a constant velocity) all fields components are implicit time-dependent functions of the type $f(\mathbf{R}(t))$. Field lines remain radial in all inertial frames of

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references and, hence, depend on the *instant* position of the charge. As a consequence, time t is not an independent variable any more in this case and enters as a parameter through space position of the charge $\mathbf{R}(t)$. Hence, the use of partial time derivatives $\partial / \partial t$, $\partial^2 / \partial t^2$ etc. (according to their formal mathematical definition) is inadequate if a function has not two or more independent variables. Nevertheless, in basic texts on classical electromagnetic theory partial time derivatives are *indiscriminately* applied even for implicit time dependent functions in the proper sense of total time derivatives. (Some clear examples will be done in the next Section discussing the use of continuity equation).

Looking back at D'Alembert's equations (15) and (16), space variable $\mathbf{R}$ should be fixed under the action of partial time derivative $\partial^2 / \partial t^2$. Fixing $\mathbf{R}(t)$, means that there is no change with time t playing the role of a parameter. Thus, partial time derivatives vanish from D'Alembert equation in the case of uniformly moving charge. Poisson's equation for four-vector $(\varphi, \mathbf{A})$ with implicit time dependence appears to be appropriate one. We especially made a detailed analysis because of confusion in conventional texts on classical electromagnetism about explicit use of Poisson's equations for uniformly moving charge (but as we have seen, they do it tacitly). It is commonly thought that only D'Alembert's equation (i.e. that only D'Alembert's operator $\Delta - \partial^2 / \partial t^2$) is relativistically invariant under Lorentz's transformations. As we will discuss later in connection with gauge invariance, Poisson's equation in four-vector representation $(\varphi, \mathbf{A})$ (as well as Poisson's differential operator Δ)can also be considered relativistically invariant when applied to implicit time-dependent potentials, reproducing all results of special relativity for inertial frames of reference. Poisson's differential operator Δ is not covariant but invariant under Lorentz's transformations. Time variable is not any more independent in this case and cannot be used for covariant representation of D'Alembert's differential operator. It is endorsed by the well-known fact that covariance is not necessary, it is only sufficient for relativistic invariance.

Thus, we can conclude that D'Alembert equations have general solutions in form of explicit time-dependent functions whereas Poisson's equations have only implicit time dependent solutions. The following question becomes obvious: *how any transition from D'Alembert and Poisson's equations is describe d in the conventional formalism*? As a matter of fact, this question has not even been asked because Poisson's equation has not been recognized as covering implicit time-dependent phenomena (it was applied exclusively in electro- and magneto-statics with no time dependence at all). This question, unexplored by the conventional approach, contains a very serious difficulty.

As we shall demonstrate below, a continuous transition between solutions of D'Alembert's and Poisson's equations, respectively, *is not mathematically ensured in classic al electromagnetism.* Based on the premises of a continuous nature of electromagnetic phenomena, one can assume that any general implicit time solution of Poisson's equation should be continuously transformed into explicit time solutions of D'Alembert's equations (and vice versa). This requirement can also be formulated as a mathematical condition on the continuity of general solutions of Maxwell's equations at every moment of time. By force of the uniqueness theorem for the second order partial differential equations, only one solution exists satisfying given initial and boundary conditions. Consequently, the continuous transition from solutions of D'Alembert's equation into solutions of Poisson's equation (and vice versa) should be ensured by the continuous transition between respective initial and boundary conditions. *This is the point where the conventional approach fails again.* Only implicit time-dependent function $f(\mathbf{R}(t))$ can be unique solution of Poisson's equations and boundary conditions for external problem are to be formulated in the infinity. On the other hand, the solution of D'Alembert's equation is an explicit time-dependent function $g(\mathbf{R}(t), t)$ since only it fits requirements of Faraday-Maxwell's electrodynamics as a physically sound solution for the notion of local (contact) field. The boundary conditions in this case are given in a finite region. It makes no sense to establish them at the infinity if it cannot be reached by any perturbations with finite spread velocity. As far as one deal with large external region, effects of boundaries are still insignificant over a small interval of time, and, therefore, it is convenient to consider the limiting problem with initial conditions for an infinite region (initial Cauchy's problem). This is how in mathematical physics areas of infinite dimensions are introduced into consideration.

Let us look carefully at the standard formulation of respective boundary-value problems in a region extending to infinity. There are three external boundary-value problems for Poisson's equation. They are known as the Dirichlet problem, Neumann problem and their combination. The mathematical formulation, for instance, for Dirichlet's boundary conditions requires finding a function $u(r)$ satisfying [7]

I. Laplace's equation $\Delta u = 0$ everywhere outside the given system of charges (currents).

II. Solution $u(r)$ is continuous everywhere in the given region and takes the given value G on the internal surface $S : u\big|_S = G$.

III. Solution $u(r)$ converges uniformly to $u(r) \to$ at infinity: $u(r) \to 0$ as. $|r| \to \infty$

The final condition (iii) is essential for a unique solution! In the case of D'Alembert's equation the standard mathematical formulation is different. Obviously, we are interested only in the problem for an infinite region (initial Cauchy's problem). So it is required to find the function $u(r(t), t)$ satisfying [5]:

a. (j) Homogeneous D'Alembert's equation everywhere outside the given system of charges (currents) for every moment of time $t \geq 0$.

b. (jj) initial conditions in all infinite regions as follows:

$$u(r,t)\big|_{t=0} = G_1(r);\ u_t(r,t)\big|_{t=0} = G_2(r).$$

The condition (iii) about the uniform convergence at infinity is not mentioned. Recall here that Cauchy's problem is considered when one of the boundaries is insignificant over all time of a process. In conventional electrodynamics it means that any perturbation with finite spread velocity will never reach the limits of the region under consideration during the time of observation.

From the conventional point of view, condition (iii) formally included into Cauchy's problem can never affect the solution and, hence, might not be taken into account seriously for selecting of adequate solutions. In fact in the context of local field, the inclusion of the condition (iii) becomes meaningless since only explicit time-dependent solutions (retarded waves with finite spread velocity) are allowed by conventional electrodynamics to solutions of D'Alembert's equation. On the other hand, we underline here that the absence of the condition (iii) for every moment of time in the standard mathematical formulation of Cauchy's initial problem does not ensure the continuous transition into external boundary-value problem for Poisson's equation and, as a result, *mutual continuity between the corresponding solutions cannot be expected by force of the uniqueness theorem*. This unambiguous mathematical fact should be considered as one of the most warning signals of possible flaws in the mathematical formalism of contemporary Maxwell's electrodynamics. The only way that seems to be obligatory to satisfy the property of continuity of electromagnetic field (in other words, to keep the continuity in transition between solutions of D'Alembert and Poisson's equations), is the inclusion of the condition (iii) for every moment of time in the standard mathematical formulation of Cauchy's initial problem. It obviously ensures the continuous transition into external boundary-value problem for Poisson's equation (and vice versa) and implies a structure of a general solution as a superposition of separate non-reducible to each other functions of the type.

$$f\left(\mathbf{R}(t)\right)+\mathrm{g}(\mathbf{r},t). \tag{17}$$

When we apply it to potentials, this statement takes the form:

$$\varphi(\mathbf{r},t)=\varphi_0\left(\mathbf{R}(t)\right)+\varphi^*(\mathbf{r},t), \tag{18}$$

$$\mathbf{A}(\mathbf{r},t)=\mathbf{A}_0\left(\mathbf{R}(t)\right)+\mathbf{A}^*(\mathbf{r},t), \tag{19}$$

Where for one charge system $\mathbf{R}(t)=\mathbf{r}-\mathbf{r}_q(t)$; r is a fixed distance from the point of observation to the origin of the reference system and $\mathbf{r}_q(t)$ is the position of the charge at the instant t.

The presence of the condition (iii) in the formulation of Cauchy's problem turns out to be meaningful for any moment of time, and the corresponding boundary conditions keep continuity in respect of mutual transformation. That makes the condition (iii) irremovable from the formulation of initial Cauchy's problem resulting in fundamental (irremovable) nature of implicit time-dependent (or *longitudinal*) components $\partial\mathbf{H}/\partial t^{*}$•• responsible for the interparticle interaction. Potentials with explicit time-dependence φ^* and $\mathbf{A}^*$ vanish in the steady-state case, leaving only implicit time-dependent functions φ_0 and $\mathbf{A}_0$ in the total potential (left-hand side of (18) and (19)). Now, contrary to the conventional approach, it is clear how the total solution φ (or $\mathbf{A}$) in left hand side of (18), (19) with explicit time dependence undergoes transformations into solution with implicit time dependence (and vice versa). Faraday-Maxwell's approach does not allow to take into account the first term in right-hand side of (18), (19) as full-value part of any general solution. Turning to the above-mentioned ambiguity at the beginning of the previous section, we see now that the novel solution in form of (18), (19) can describe the change of electric field component along the *X*-axis at any distance and at any time. It casts doubts on the general belief that Lienard-Wiechert potentials (as only explicit time-dependent solutions of D'Alembert's equations for Cauchy's problem) should be considered as unique general solutions to Maxwell's equations regardless the context of boundary conditions. In fact, Lienard and Wiechert formulated the initial Cauchy problem for electromagnetic components several years before the appearance of Einstein's principle of relativity. Thus, a priori imposed boundary conditions were not assumed to have adequate relativistic properties. This is another open question in the conventional approach whether relativistic requirements should be reflected in the mathematical formulation of the initial boundary problem. In this respect, we only stress that additional condition (iii) is such an invariant because it is irremovable and unchangeable in every frame of reference.

Let us consider again a pair of uncoupled inhomogeneous D'Alembert's equations (15), (16) with initial conditions (j), (jj) and (iii). F or some purposes, it is convenient to decompose (15), (16) into two pairs of second order differential equations for each component of general solution of (15), (16):

$$\Delta\varphi_0=-4\pi\tilde{n}(\mathbf{r},t), \tag{20}$$

$$\Delta\mathbf{A}_0=-\frac{4\pi}{c}\mathbf{j}(\mathbf{r},t) \tag{21}$$

and

$$\Delta\varphi^*-\frac{1}{c^2}\frac{\partial^2\varphi^*}{\partial t^2}=0, \tag{22}$$

$$\Delta\mathbf{A}^*-\frac{1}{c^2}\frac{\partial^2\mathbf{A}^*}{\partial t^2}=0, \tag{23}$$

with initial and boundary conditions given, for instance, in the case of electric potential. The equation (20), apart from (iii), is supplemented by

$$\varphi_0(\mathbf{r})\Big|_S=G \tag{24}$$

Whereas (22) has to be added with

$$\varphi^*(\mathbf{r},t)\Big|_{t=0}=G_1-\varphi_0(\mathbf{r})\Big|_{t=0}, \tag{25}$$

$$\varphi^*_\tau(\mathbf{r},t)\Big|_{t=0}=G_2-\frac{d}{dt}\varphi_0(\mathbf{r})\Big|_{t=0}. \tag{26}$$

In the theory of differential equations any complete solution of (15), (16) consists of a general solution of homogeneous D'Alembert's equation plus some particular solution of the inhomogeneous one. Thus, we can assume that the same procedure can be applied to its equivalent formulation in form (20)-(23). On one hand, a complete solution should be formed by two independent general solutions satisfying homogeneous Poisson's and homogeneous wave equations, respectively, and, on the other hand, it has to include one particular solution (as a linear combination of non-reducible components (18), (19), satisfying inhomogeneous D'Alembert's equations (15,16). Relationship between both components (*longitudinal* and transverse) of electromagnetic field is guided by (25) and (26) and is contained in the particular solution of inhomogeneous D'Alembert's equations. A more comprehensive study of the matter will be done elsewhere.

Thus, the initial set of Maxwell's equations has been decomposed into two pairs of equations with independent general solutions for each pair that are coupled only through the partial solution of the whole set of equations (20)-(23) or (15), (16). The first pair (20), (21) manifests the instantaneous and longitudinal aspect of electromagnetic interactions (action-at-a-distance) while the second one (22), (23) characterizes explicit time-dependent phenomena related to the propagation of transverse waves (light, radiation etc.). It is obvious thus that Helmholtz's basic ideas are fundamentally compatible with Maxwell's equations. The potential separation (18), (19) implies the same procedure with respect to the field strengths,

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0\left(\mathbf{R}(t)\right) + \mathbf{E}^*(\mathbf{r},t), \tag{27}$$

$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0\left(\mathbf{R}(t)\right) + \mathbf{B}^*(\mathbf{r},t), \tag{28}$$

Where $\mathbf{E}_0$ and $\mathbf{B}_0$ are instantaneous *longitudinal* fields.

To finish this Section we would like to mention that Villecco's independent analysis endorsed our claims on discontinuity problem in the classical electromagnetic theory. He found that [8]: " *...the transition between two different states of uniform velocity via an intermediate state of acceleration results in a type of discontinuity in functional form: Though no known law is violated in this processes, there is a sense of intrinsic continuity which is nevertheless violated...*"

Mathematical inconsistencies in the formulation of Maxwell-Lorentz equations for one charge system

To understand what is happening inside a dipole with transfer of energy and momentum from one dipole charge to another, we must first understand what is happening to one charge in terms of the conventional electrodynamics, when it moves.

Let us come back again to the original set of Maxwell's equations (10)-(13) for the reference system at rest supplemented by the continuity equation

$$\frac{\partial \tilde{n}}{\partial t} + \nabla\cdot\mathbf{j} = 0\,. \tag{29}$$

In the phenomenological theory of electromagnetism the hypothesis about the continuous nature of the medium was one of the foundations of Maxwell's theoretical scheme. This point of view succeeded in uniting so many electromagnetic phenomena without the necessity to consider a specific structure of matter. Nevertheless, a macroscopic character of the charge conception defines all well-known limitations on Maxwell's theory. For instance, the system of equations (10)-(13) in a steady state approximation corresponds to a quite particular case of continuous and closed conduction currents (motionless as a whole).

In 1895, the theory was extended by Lorentz for a system of charged particles moving in vacuum. Since then it has been widely assumed that the same basic laws are valid microscopically as it is macroscopically in the case of original Maxwell's equations. This means that in Lorentz form all macroscopic values of charge and current densities have to be substituted by their microscopic values. Let us write explicitly the Lorentz field equations for one charged point particle moving in vacuum [2]:

$$\nabla\cdot\mathbf{E} = 4\pi q\delta\left(\mathbf{r}-\mathbf{r}_q(t)\right), \tag{30}$$

$$\nabla\cdot\mathbf{H} = 0, \tag{31}$$

$$\nabla\times\mathbf{H} = \frac{4\pi}{c}q\mathbf{v}\delta\left(\mathbf{r}-\mathbf{r}_q(t)\right) + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \tag{32}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \tag{33}$$

Where $\mathbf{r}_q(t)$ is the coordinate of a charge at the moment of time t.

In order to achieve a complete description of a system consisting of fields and charges in the framework of electromagnetic theory, Lorentz supplemented (30)-(33) by the equation of motion:

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}\left(\mathbf{v}\times\mathbf{H}\right), \tag{34}$$

where $\mathbf{p}$ is the momentum of the particle.

The equation of motion (34) introduces an expression for the mechanical force known as Lorentz force which in the electron theory formulated by Lorentz has a clear axiomatic and empirical status. Later on we shall discuss some disadvantages related with the adopted status of the Lorentz force conception.

Macroscopic Maxwell's equations (10)-(13) may be obtained now from Lorentz's equations (30)-(33) by some statistical averaging process, using the structure of material media. The mathematical language for equations (30)-(33) is nowadays widely accepted in the conventional classical electrodynamics. However, there is an ambiguity in the application of these equations to the case of one uniformly moving charge. A simple charge translation in space produces alterations of field components. Nevertheless, they cannot be treated in terms of Maxwell's displacement currents. Strictly speaking, in this case all Maxwell's displacement currents proportional to $\partial\mathbf{E}/\partial t$ and $\partial\mathbf{H}/\partial t$ vanish from (32), (33). This statement can be reasoned in two different ways:

1. $\partial\mathbf{E}/\partial t = 0$ and $\partial\mathbf{H}/\partial t = 0$, since all field components of one uniformly moving charge are implicit time-dependent functions (time does not enter as an independent parameter but only through space variable) so that from the mathematical standpoint only total time derivative makes sense in this case whereas partial time derivative turns out to be not adequate (time and distance are not independent variables); 2. A non-zero value of $\partial\mathbf{E}/\partial t^*$ and $\partial\mathbf{H}/\partial t^*$ would imply a local variation of fields in time regardless any change in the position of the charge (space coordinate is fixed when partial time derivative is taken) and, hence, would imply the propagation of those local variations in form of transverse electromagnetic waves.

This would strongly contradict the well-established in special relativity fact that one uniformly moving charge does not produce any electromagnetic radiation at all.

Thus, a mathematically rigorous interpretation of (32), (33) in the case of a charge moving with a constant velocity leads to the following conclusion: in a charge-free space the value of $^*\partial\mathbf{E}/\partial t = 0$ and, therefore, the value of $\nabla\times\mathbf{H}$ is also equal to zero in free space.

$$\nabla\times\mathbf{H}=\frac{4\pi}{c}q\mathbf{v}\delta\left(\mathbf{r}-\mathbf{r}_q(t)\right). \qquad (35)$$

On the other hand, field components of one uniformly moving charge can be treated exactly in the framework of Lorentz's transformations. Therefore, for any purpose exact relativistic expressions for electric and magnetic fields and potentials should be applied [2].

$$\mathbf{E}=q\frac{\left(1-\beta^2\right)\left(\boldsymbol{R}-R\boldsymbol{\beta}\right)}{\left(\boldsymbol{R}-R\boldsymbol{\beta}\right)^3}, \qquad (36)$$

$$\mathbf{H}=\frac{1}{c}\left(\mathbf{v}\times\mathbf{E}\right), \qquad (37)$$

Where $\boldsymbol{\beta}=\mathbf{v}/c$.

Thus, we arrive here at the important conclusion: generally speaking, according to special relativity theory the value of $\nabla\times\mathbf{H}$ is not equal to zero in any point out of moving charge and takes a well-defined value.

$$\nabla\times\mathbf{H}=\frac{1}{c}\left(\nabla\times\left(\mathbf{v}\times\mathbf{E}\right)\right). \qquad (38)$$

For instance, this gives immediately a non-zero value of $\nabla\times\mathbf{H}$ along the direction of motion (X-axis):

$$\nabla\times\mathbf{H}\left(x,x>x_0\right)=q\frac{2\boldsymbol{\beta}\left(1-\beta^2\right)}{(1-\beta)^3(x-x_0)^3}. \qquad (39)$$

The conflict with the previous statement of the equation (31) is inevitable. In order to obtain adequacy between the set of field equations (30)-(33) and their relativistic solutions in the case of uniformly moving charge, it is necessary to consider an additional term like that considered in (38). As will be shown in continuation, this assumption for static and quasi-static fields is a supplement of Maxwell's displacement currents introduced for explicitly time varying fields (explanation of the light as the propagation of transverse electromagnetic waves).

As it is well-known, the necessity of Maxwell's displacement current was realized on the basis of the following formal reasoning. In order to make equation (8) consistent with the electric charge conservation law in form of continuity equation (29), Maxwell supplemented (12) with an additional term. However, for stationary processes, as we already have seen, this term disappears and equation (12) becomes consistent only with closed (or continuous going off to infinity) currents.

$$\nabla\cdot\mathbf{j}=0. \qquad (40)$$

It is also a direct consequence of continuity equation (29) in any stationary state when all magnitudes have to be treated as implicit time-dependent functions. Thereby, we meet here another difficulty of Lorentz's equations: uniform movement of a single charged particle (as an example of open steady current), generally speaking, does not satisfy the limitations imposed by (40). It implies some additional term to be taken into account in (40) to fulfil Maxwell's hypothesis on the circuital character of total currents (conduction plus displacement currents).

Let us have a close look on the continuity equation and its conventional interpretation. In developing the mathematical formalism of his theory Maxwell adopted Faraday's idea of field tubes for electric and magnetic fields as well as for electric charge flow (conduction currents). As a consequence, in accordance with hydrodynamics language, the continuity equation was accepted as valid to express the hypothesis that a net sum of electric charge could not be annihilated. In this case, the continuity equation reproduces the charge conservation law in the given fixed volume V.

$$\frac{dQ}{dt}=\iiint_V\left\{\frac{\partial\varrho}{\partial t}+\nabla\cdot\mathbf{j}\right\}dV=0 \qquad (41)$$

Or in the form of a differential equation

$$\frac{\partial\varrho}{\partial t}+\nabla\cdot\mathbf{j}=0,\ \left(\frac{dQ}{dt}=0\right). \qquad (42)$$

It should be remarked that equation (42) describes exclusively the conservation but not the change of the amount of charge (or matter) in the given volume V. In many scientific writings on electromagnetic theory there is no clear distinction between these two aspects. If one wants to describe the change of something in the given volume V, the equation (41) should be replaced by a balance equation (see, for instance, [9])

$$\frac{dQ}{dt}=\frac{d}{dt}\iiint_V\varrho dV=-\iint_S\nabla\cdot\mathbf{j}dS, \qquad (43)$$

Here $\mathbf{j}$ is a total current of electric charges through a surface S that bounds the given volume V. In the mathematical language common to all physical theories it means that the rate of increase in the total quantity of electrostatic charge within any fixed volume mathematical language common to all physical theories it means that the rate of increase in the total quantity of electrostatic charge within any fixed volume V is equal to the excess of the influx over the efflux of current through a closed surface S. On contracting the surface to an infinitesimal sphere around a point one can arrive at the differential equation [9].

$$\frac{\partial\varrho}{\partial t}+\nabla\cdot\mathbf{j}=0,\ \left(\frac{dQ}{dt}\neq0\right). \qquad (43)$$

The balance equation (43) covers the continuity equation (42) as a particular case in which the amount of something (charge or matter) is kept constant in V during the course of time. Earlier we mentioned that a single charge in motion, generally speaking, could not be treated in terms of the continuity equation (42). When the particle leaves the given volume, it violates locally the charge conservation, invalidating the continuity equation (42). Instead of it the balance equation (43) has to be used. One simple method to prove that is to consider again the example of point-charge moving with a constant velocity. In particular, the charge density is assumed to have implicit time dependence as follows.

$$\varrho\left(\mathbf{r},\mathbf{r}_q(t)\right)=q\delta\left(\mathbf{r}-\mathbf{r}_q(t)\right), \qquad (44)$$

Where $\mathbf{r}$ is a fixed distance from the point of observation to the origin of the reference system at rest; $\mathbf{r}_q(t)$ and $\mathbf{v}_q=d\mathbf{r}_q/dt$ are the distance and the velocity of the charge at the instant.

It is easy to show that the total density derivative with respect to time consist of the convection term only , since time enters in equation (44) as a parameter : $\left(\partial\varrho/\partial t=0\right)$

$$\frac{d\varrho}{dt}=\frac{\partial\varrho}{\partial t}+\left\{\frac{d}{dt}\left(\mathbf{r}-\mathbf{r}_q(t)\right)\right\}\nabla\varrho=-\left(\mathbf{v}_q\cdot\nabla\varrho\right). \qquad (45)$$

Thus, the balance equation for a single charged particle is fulfilled directly:

$$-\left(\mathbf{v}_q\cdot\nabla\varrho\right)+\nabla\left(\varrho\mathbf{v}_q\right)=-\left(\mathbf{v}_q\cdot\nabla\varrho\right)+\left(\mathbf{v}_q\cdot\nabla\varrho\right)=0. \qquad (46)$$

The next step is to analyze equation (45) in terms of Maxwell's hypothesis in respect to the circuital character of the total electric current (including displacement current). In other words, the total current of one uniformly moving charge has to be formed by two contributions: the motion of the charge itself (conduction current) and displacement current in outer space:

$$\nabla\cdot\left(\mathbf{j}_{\text{cond}}+\mathbf{j}_{\text{displ}}\right)=0, \qquad (47)$$

Where $\mathbf{j}_{\text{cond}}$ and $\mathbf{j}_{\text{displ}}$ are conduction and displacement currents, respectively.

Thus, we can rewrite (43) in the form of equation (47).

$$\nabla\cdot\mathbf{j}_{\text{displ}}=\frac{d\varrho}{dt}=\frac{d}{dt}\left(\frac{1}{4\pi}\nabla\cdot\mathbf{E}\right). \qquad (48)$$

It may be easily verified that two field operations ∇ and d/dt are completely interchangeable in (48). Thus, for general motion of the charge when one can disregard its size, Maxwell's condition on a total current takes the following form (see for the sake of comparison the formula (45)) taking into account the standard expansion of the total time derivative (the index for ∇ indicates which of the functions it operates on):

$$\nabla\cdot\mathbf{j}_{\text{displ}}=\frac{1}{4\pi}\left(\nabla\cdot\left\{\frac{\partial\mathbf{E}}{\partial t}-(\mathbf{v}\cdot\nabla_r)\mathbf{E}-(\mathbf{a}\cdot\nabla_v)\mathbf{E}-\ldots\right\}\right), \qquad (49)$$

Here $\mathbf{a}$ is acceleration and further terms correspond to derivatives of non-uniform acceleration. So far we have made use of the formal mathematical approach without any physical interpretation. More specifically, in calculating the full time derivative of $\mathbf{E}$, the convective term (second right-hand term in (49)) should be considered as implicit time-dependent (time variable is fixed when space partial derivative is taken) in agreement with the mathematical definition of partial derivatives. In mathematical language it means that all field alterations produced by a simple charge translation (convective part of the total derivative) take place at the same time in every space point (i.e. instantaneously). This interpretation has no precedents in conventional classical electrodynamics for the case of arbitrary motion whereas for uniformly moving charge this description is the only possible formalism (in special relativity field lines of uniformly moving charge remain radial, i.e. exhibit no retardation in respect to the space position of the charge). Turning back to (49), it is clear that the first right-hand term with partial time derivative describes explicit time-dependent phenomena. Thus, in the same way as it was independently concluded in the Section 2, all field components can be split up into two independent classes with explicit $\mathbf{E}^*$ and implicit $\mathbf{E}_0$ time dependencies, respectively:

$$\frac{d\mathbf{E}}{dt}=\frac{\partial\mathbf{E}^*}{\partial t}-\left(\mathbf{v}\cdot\nabla_r\right)\mathbf{E}_0-\left(\mathbf{a}\cdot\nabla_v\right)\mathbf{E}_0-\ldots \qquad (50)$$

A general expression of full displacement current is then taken by the formula:

$$\mathbf{j}_{\text{displ}}=\frac{1}{4\pi}\frac{\partial\mathbf{E}^*}{\partial t}-\frac{1}{4\pi}\left(\mathbf{v}\cdot\nabla_r\right)\mathbf{E}_0-\frac{1}{4\pi}\left(\mathbf{a}\cdot\nabla_v\right)\mathbf{E}_0-\ldots \qquad (51)$$

Let us stress here one subtle point which will be indispensable in the following discussion of relativistic invariance properties of the Helmholtz-type approach. The derivation of (50) has considered the partial time derivative to be independent from the space derivative in full agreement with the mathematical formalism of partial derivatives. Thus, the time parameter of implicit time-dependent components (let us call it t) comes into consideration as an afterthought through the space variable $R(t)$ and, therefore, can be, in principle, considered as independent from the time variable of explicit time-dependent components (in special relativity this is the so-called proper time τ). As we will discuss later, special relativity does not distinguish these two time dependences and tacitly implies $t=\tau$ that leads to the Lorentz invariance of electromagnetic field components. In order to come back to the previous discussion of the displacement current concept, let us remind that our initial aim was to find a reasonable form for Maxwell's circuital condition (50). It would allow relating field alterations in free space produced by one moving charge with the Maxwell conception of displacement current. From the standpoint of conventional classical electrodynamics, the first term represents the well-known Maxwell displacement current coming up only in non-steady processes whereas the second term can be interpreted only as quasistationary due to its dependence on a charge translation in space (with time as implicit parameter). Further, we will call that term as "*convection displacement current*". By the same token, the third right-hand term is due to uniform acceleration and could be called "*uniform acceleration displacement current*" etc.

The above results motivate an important extension of displacement current concept. First, it postulates the circuital character of the total electric current as it was originally assumed by Maxwell. Second, it permits to fulfil the circuital condition for non-steady as well as for steady processes (static and quasistatic fields), contrary to the conventional approach. Let us give an equivalent mathematical expression of the convection displacement current (in the case of single charged particle):

$$\frac{1}{c}\left(\mathbf{v}\cdot\nabla\right)\mathbf{E}=\frac{1}{c}\mathbf{v}\left(\nabla\cdot\mathbf{E}\right)-\frac{1}{c}\left(\nabla\times\left(\mathbf{v}\times\mathbf{E}\right)\right). \qquad (52)$$

Accordingly, for our purpose we need to remind that in the right-hand side of equation (32) the total current $\left(\mathbf{j}_{\text{tot}}=\mathbf{j}_{\text{cond}}+\mathbf{j}_{\text{displ}}\right)$ must be considered as:

$$\nabla\times\mathbf{H}=\frac{4\pi}{c}q\mathbf{v}\delta\left(\mathbf{r}-\mathbf{r}_q(t)\right)+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}-\frac{1}{c}\mathbf{v}\left(\nabla\cdot\mathbf{E}\right)+\frac{1}{c}\left(\nabla\times\left(\mathbf{v}\times\mathbf{E}\right)\right)+\ldots \qquad (53)$$

For the sake of simplicity we omit acceleration and other expansion terms in this general formula but they are tacitly implied. This approach allows the treatment of equation (33) in the same way as (32):

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}+\frac{1}{c}\left(\mathbf{v}\cdot\nabla\right)\mathbf{H}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}+\frac{1}{c}\mathbf{v}\left(\nabla\cdot\mathbf{H}\right)-\frac{1}{c}\left(\nabla\times\left(\mathbf{v}\times\mathbf{H}\right)\right)+\ldots \qquad (54)$$

Turning back to the beginning of this Section we note now that for uniform motion $\nabla\times\mathbf{H}$ is defined by (53) in every space point out of the charge in the expected way (see (38)). As a final remark, the set of equations (30), (31) and (53), (54) can be regarded as a generalized form of Maxwell-Lorentz system of field equations. In the next section they will be compared with modified Maxwell-Hertz equations extended on one charge system.

Reconsidered Maxwell-Hertz theory and relativistic invariant formulation of generalized Maxwell's equations

Independently of Heaviside, the problem of the modification of Maxwell's equations for bodies in motion was posed by Hertz in his attempts to build up a comprehensive and consistent electrodynamics [8,9]. A starting point of that approach was the fundamental character of Faraday's law of induction represented for the first time by Maxwell in the form of integral equations.

$$\oint_C \mathbf{H}dl = \frac{4\pi}{c}\iint_S \mathbf{j}dS + \frac{1}{c}\frac{d}{dt}\iint_S \mathbf{E}dS \,, \qquad (55)$$

$$\oint_C \mathbf{E}dl = -\frac{1}{c}\frac{d}{dt}\iint_S \mathbf{H}dS \,, \qquad (56)$$

Where C is a contour, S is a surface bounded by C.

In qualitative physical language Faraday's observations had been expressed in form of the following statement: *the effect of magnetic induction in the circuit* C *takes place always with the change of the magnetic flux through the surface S regardless whether it relates to the change of intensity of adjacent magnet or occurs due to the relative motion*. Moreover, Faraday established that the same effect was detected in a circuit at rest as well as in that in motion. The latter fact provided the principal basis of Hertz's relativity principle based on Galileo invariance. In order to avoid details of Hertz's original investigations [10,11], let us only note its similarity with the traditional non-relativistic treatment of the integral form of Faraday's law [12]. Namely, if the circuit C is moving with a velocity v in some direction, the total time derivative in (53), (56) must take into account this motion (convection derivative) as well as the flux changes with time at a point (partial time derivative) [12].

$$\oint_C \mathbf{E}dl = -\frac{1}{c}\frac{d}{dt}\iint_S \mathbf{H}dS = -\frac{1}{c}\left\{\frac{\partial}{\partial t}+(\mathbf{v}\cdot\nabla)\right\}\iint_S \mathbf{H}dS \,, \qquad (57)$$

Where S is any surface bounded by circuit C, moving together with a medium.

This approach is valid only for non-relativistic consideration and leads to Galilean field transformation (46). In Hertz's theory any motion of the ether relative to the material particles had not been taken into account, so that the moving bodies were regarded simply as homogeneous portions of the medium distinguished only by special values of electric and magnetic constants. Among the consequences of such assumption, Hertz saw the necessity to move the surface of integration in equations (55), (56) at the same time with the moving medium. Thus the generation of a magnetic (or electric) force within a moving dielectric was calculated with implicit use of Galilean invariance in equation (57) unless one makes any additional assumptions on the special character of transformations in a moving frame of reference. Recently, T. Phipps Jr. again drew attention to the failure of Maxwell's equations in partial time derivative to describe first-order effects related to convective terms of total time derivatives [13,14]. He proposed to revive Hertz's Galilean-invariant version of Maxwell's theory written in total time derivatives. He only differs from Hertz's own interpretation of the velocity parameter. However, in this review we shall show how total time derivatives can be compatible with the requirements of special relativity in inertial frames of reference.

Let us now examine the case of a point source of electric and magnetic fields. In order to abstain from the use of moving contour C and surface S that implies a priori application of some relativity principle (Galileo's or Einstein's), we limit our consideration to a fixed region (S and S are at rest) whereas the source is moving through a free space. According to Faraday's law, there must be an electromotive force in the contour C due to the flux changes with time and convection derivatives simultaneously. Using the mathematical language for total time derivatives, we arrive at the expression analogous to the differential form (50).

$$\frac{d\Phi}{dt}=\frac{\partial\Phi^*}{\partial t}-(\mathbf{v}_S\cdot\nabla_r)\Phi_0-(\mathbf{a}_S\cdot\nabla_v)\Phi_0-\ldots, \qquad (58)$$

Making use of the definitions:

$$\Phi_0^E=\iint_S \mathbf{E}_0(\mathbf{r}-\mathbf{r}_S(t))dS, \text{ or,}$$

$$\Phi_0^B=\iint_S \mathbf{B}_0(\mathbf{r}-\mathbf{r}_S(t))dS \qquad (59)$$

and

$$\mathbf{\Phi}^{*(E)}=\iint_S \mathbf{E}^*(\mathbf{r},t)dS, \text{ or,}$$

$$\mathbf{\Phi}^{*(B)}=\iint_S \mathbf{B}^*(\mathbf{r},t)dS, \qquad (60)$$

Where $\mathbf{r}$ is a fixed distance from the point of observation to the origin of the reference systems at rest; $\mathbf{r}_S(t)$, $\mathbf{v}_S = d\mathbf{r}_S/dt$, $\mathbf{a}_S = d\mathbf{v}_S/dt$ are the distance, the instant velocity, the instant acceleration of the electric (or magnetic) field source.

For the sake of simplicity, we can conserve for the present the same denomination of field flux in two independent parts of total time derivative (59), taking into account additional (fixed space and fixed time) conditions, respectively, in the following expression:

$$\frac{d}{dt}\Phi = \left\{\frac{\partial}{\partial t}-(\mathbf{v}_S\cdot\nabla_r)-(\mathbf{a}_S\cdot\nabla_v)-\ldots\right\}\Phi. \qquad (61)$$

Using a well-known representation for the convection part in equation (59),

$$(\mathbf{v}\cdot\nabla)\iint_S \mathbf{E}dS = \iint_S \mathbf{v}(\nabla\cdot\mathbf{E})dS + \iint_S \nabla\times(\mathbf{E}\times\mathbf{v})dS \,, \qquad (62)$$

We obtain an alternative form of Maxwell's integral equations (55), (56) for a moving electric charge in the reference system at rest.

$$\oint_C \mathbf{H}dl = \frac{4\pi}{c}\iint_S \mathbf{j}dS + \frac{1}{c}\iint_S\left\{\frac{\partial \boldsymbol{E}}{\partial t}-\mathbf{v}(\nabla\cdot\mathbf{E})-\nabla\times(\mathbf{E}\times\mathbf{v})-\ldots\right\}dS \,, \qquad (63)$$

$$\oint_C \mathbf{E}dl = -\frac{1}{c}\iint_S\left\{\frac{\partial \boldsymbol{E}}{\partial t}+\nabla\times(\mathbf{v}\times\mathbf{H})+\ldots\right\}dS \,. \qquad (64)$$

Here we omit, for the sake of simplicity, acceleration and other expansion terms in general formula but they, of course, are tacitly implied.

Before going on to a more general consideration of a large number of sources, it is worth to draw attention that we arrived to the most compact differential form of Maxwell- Hertz equations in the reference system at rest [15].

$$\nabla \cdot \mathbf{E} = 4\pi\varrho \,, \tag{65}$$

$$\nabla \cdot \mathbf{H} = 0 \,, \tag{66}$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\varrho\mathbf{v} + \frac{1}{c}\frac{d\mathbf{E}}{dt} \,, \tag{67}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{d\mathbf{H}}{dt} \,, \tag{68}$$

Where the total time derivative of any vector field value $\mathbf{E}$ (or $\mathbf{H}$) is,

$$\frac{d\mathbf{E}}{dt} = \frac{\partial\mathbf{E}}{\partial t} - (\mathbf{v}\cdot\nabla)\,\mathbf{E} - (\mathbf{a}\cdot\nabla_v)\,\mathbf{E} - \ldots \tag{69}$$

The above-mentioned form (65)-(68) was for the first time admitted by Hertz for electrodynamics of bodies in motion [10,11]. It was the covering theory for Maxwell's original approach which became the limit case of motionless medium (a reference system at rest) when values of instant velocity $\mathbf{v}$, instant acceleration a etc. tend to zero in (69) leaving only partial time derivatives in agreement with (10)-(13). The difference of the present approach [15] with Hertz's covering theory (and with Phipps' neo-Hertzian approach [13,14]) consists in the definition of the total time derivative (66) for a medium at rest (not in motion with the possible implication of Galilean invariance). Below we shall demonstrate that the set (65)-(68) possesses invariance properties in any inertial frame of reference.

There is no difficulty in extending this approach to a many particle system, assuming the validity of the electrodynamics superposition principle. This extension is important in order to find out whether the generalized microscopic field equations cover the original (macroscopic) Maxwell's theory as a limiting case. To do so one ought to take into account all principal restrictions of Maxwell's equations (10)-(13) which deal only with a continuous and closed (or going off to infinity) conduction currents. They also have to be motionless as a whole (static tubes of charge flow), admitting only the variation of current intensity.

Under these assumptions, it is quite easy to show that the total (macroscopic) convection and others displacement currents are cancelled by itself by summing up all microscopic contributions,

$$\begin{gathered}\sum_i(\mathbf{v}_i\cdot\nabla)\mathbf{E}_i+(\mathbf{a}_i\cdot\nabla_v)\mathbf{E}_i+\ldots \\ \sum_i(\mathbf{v}_i\cdot\nabla)\mathbf{H}_i+(\mathbf{a}_i\cdot\nabla_v)\mathbf{H}_i+\ldots\end{gathered} \tag{70}$$

In other words, every additional terms in (53), (54) (as well as in (63), (64) disappears and we obtain the original set of Maxwell macroscopic equations (10)-(13) for continuous and closed (or going off to infinity) conduction currents as a valid approximation. To conclude this part we would like to note that the set of equations (63), (64) can be called as modified Maxwell-Hertz's equations extended to one charge system. It is easy to see that in this form they are completely equivalent to modified Maxwell-Lorentz equations (53), (54) obtained with the help of the balance equation. Thus, differential and integral approaches to extend the original Maxwell theory lead to the same result.

Let us write once again the generalized form of Maxwell-Lorentz equations explicitly for a single moving particle that is a source of electric and magnetic fields simultaneously,

$$\nabla \cdot \mathbf{E} = 4\pi\varrho \,, \tag{71}$$

$$\nabla \cdot \mathbf{H} = 0 \,, \tag{72}$$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\varrho\mathbf{v} + \frac{1}{\mathrm{c}}\left\{\frac{\partial\mathbf{E}}{\partial t} - (\mathbf{v}\cdot\nabla)\mathbf{E} - (\mathbf{a}\cdot\nabla_v)\mathbf{E} - \ldots\right\} , \tag{73}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t} - \frac{1}{\mathrm{c}}\nabla\times(\mathbf{v}\times\mathbf{H}) - \frac{1}{\mathrm{c}}(\mathbf{a}\cdot\nabla_v)\,\mathbf{H} - \ldots \tag{74}$$

At the same time with the balance equation,

$$\frac{d\varrho}{dt} + (\nabla\cdot\varrho\mathbf{v}) = 0. \tag{75}$$

Splitting up field components into explicit and implicit time-dependent contributions $\mathbf{E}^*\left(\mathbf{H}^*\right)$ and $\mathbf{E}_0\left(\mathbf{H}_0\right)$, respectively , the basic field equations (73), (74) can be rewritten as follows:

$$\nabla \times \mathbf{H} = \frac{4\pi}{c}\varrho\mathbf{v} + \frac{1}{\mathrm{c}}\left\{\frac{\partial\mathbf{E}^*}{\partial t} - (\mathbf{v}\cdot\nabla)\mathbf{E}_0 - (\mathbf{a}\cdot\nabla_v)\mathbf{E}_0 - \ldots\right\} , \tag{76}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}^*}{\partial t} - \frac{1}{\mathrm{c}}\nabla\times(\mathbf{v}\times\mathbf{H}_0) - \frac{1}{\mathrm{c}}(\mathbf{a}\cdot\nabla_v)\,\mathbf{H}_0 - \ldots, \tag{77}$$

Where the total field values have two independent parts,

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}^* = \mathbf{E}_0\left(\mathbf{r}-\mathbf{r}_q(t)\right) + \mathbf{E}^*(\mathbf{r},t) \,, \tag{78}$$

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}^* = \mathbf{H}_0\left(\mathbf{r}-\mathbf{r}_q(t)\right) + \mathbf{H}^*(\mathbf{r},t) \,. \tag{79}$$

Here we note that implicit time-dependent field components $\mathbf{E}_0$ and $\mathbf{H}_0$ depend only on the point of observation and on the source position at an instant whereas time varyingfields $\mathbf{E}^*$ and $\mathbf{H}^*$ depend explicitly on time at a fixed point. The separation procedure may be similarly extended to the electric and magnetic potentials introduced as

$$\mathbf{E} = -\nabla\varphi \,, \quad \mathbf{H} = \nabla\times\mathbf{A} \,, \tag{80}$$

Where

$$\varphi = \varphi_0 + \varphi^* , \quad \mathbf{A} = \mathbf{A}_0 + \mathbf{A}^* . \tag{81}$$

Let us establish invariance of field equations in total time derivatives. As far as in special relativity the invariance is looking for inertial frames of reference moving with a constant velocity $\mathbf{v}$, then in total time derivative expansion we should omit all acceleration and higher order terms. Thus, using definitions (80), (81) we obtain from equation (77) that

$$\mathbf{E} = -\nabla\varphi - \frac{1}{\mathrm{c}}\frac{\partial\mathbf{A}^*}{\partial t} - \frac{1}{\mathrm{c}}(\mathbf{v}\times\mathbf{H}_0) \,. \tag{82}$$

Separation of implicit time-dependent from explicit time-dependent components in (82) is straightforward

Using this separation we obtain two second order differential equations for total potentials (81)

$$\mathbf{E}_0 = -\nabla\varphi_0 - \frac{1}{c}(\mathbf{v}\times\mathbf{H}_0), \tag{83}$$

Where $\mathbf{E}^* = -\nabla\varphi^* - \frac{1}{c}\frac{\partial\mathbf{A}^*}{\partial t}$.

$$\Delta\mathbf{A} = \frac{4\pi}{c}\varrho\mathbf{v} + \mathbf{F}, \tag{84}$$

$$\Delta\varphi = -4\pi\varrho + \Phi, \tag{85}$$

$$\mathbf{F} = \nabla\left\{\nabla\cdot\left(\mathbf{A}_0 + \mathbf{A}^*\right)\right\} - \frac{1}{c}(\mathbf{v}\cdot\nabla)\nabla\varphi_0 + \frac{1}{c}\frac{\partial\nabla\varphi^*}{\partial t} + \frac{1}{c^2}\frac{\partial^2\mathbf{A}^*}{\partial t^2}, \tag{86}$$

$$\Phi = -\frac{1}{c}\left(\nabla\cdot\frac{\partial\mathbf{A}^*}{\partial t}\right). \tag{87}$$

The second term in (86) can be easily transformed using mathematical operations of field theory,

$$(\mathbf{v}\cdot\nabla)\nabla\varphi_0 = \nabla(\mathbf{v}\cdot\nabla\varphi_0) - \mathbf{v}\times(\nabla\times\nabla\varphi_0). \tag{88}$$

Since $\nabla\times\nabla(\ldots)$ is always equal to zero, we can rewrite $\mathbf{F}$ in a new form,

$$\mathbf{F} = \nabla\left[\left\{\nabla\cdot\mathbf{A}_0 - \frac{1}{c}\mathbf{v}\cdot\nabla\varphi_0\right\} + \left\{\nabla\cdot\mathbf{A}^* + \frac{1}{c}\frac{\partial\varphi^*}{\partial t}\right\}\right] + \frac{1}{c^2}\frac{\partial^2\mathbf{A}^*}{\partial t^2}. \tag{89}$$

The principal feature of (89) consists in the fact that all implicit and explicit time-dependent components of total electric and magnetic potentials enter independently and, therefore, can be characterized by respective gauge conditions,

$$\nabla\cdot\mathbf{A}_0 - \frac{1}{c}\mathbf{v}\cdot\nabla\varphi_0 = 0, \tag{90}$$

$$\nabla\cdot\mathbf{A}^* + \frac{1}{c}\frac{\partial\varphi^*}{\partial t} = 0. \tag{91}$$

Lorentz's gauge (91) is applicable now only for explicit time-dependent potentials and is invariant under Lorentz's transformations. It suggests that the proper time τ (let us call here the time τ variable of explicit time-dependent components in the entire spirit of the special relativity theory) for two inertial frames moving with respect to each other are related by an imaginary rotation in space-time. The amount of rotation depends on the relative velocity. Implicit time-dependent potentials turn out to be related through the novel gauge (90) which covers the well-known relationship between the components of electric and magnetic field potentials of uniformly moving charge [2],

$$\mathbf{A}_0 = \frac{\mathbf{v}}{c}\varphi_0. \tag{92}$$

Strictly speaking, this relationship is true for Galilean as well as for Lorentz's transformations. The difference is attributed to a mathematical formulation of potentials in a new frame of reference. For instance, the Lorentz transformation corresponds to a rotation in the space-time plane whereas the Galilean one leaves $\mathbf{A}_0$ and φ_0 unchanged, for it is assumed that no operation can rotate the time axis into the space axis or vice versa. For Galilean invariance, the time direction is supposed to be the same for all inertial frames of reference.

The expression (90) and all physically possible transformations based on it, do not involve explicitly any time dimension. The time t here can be added as an afterthought (a parameter describing the space coordinate $\mathbf{R}(t)$). In above discussion of full time derivative we noted that time variable τ (for explicit) and time parameter t (for implicit time behaviours) are, generally speaking, independent. If we assume, as they do it tacitly in special relativity with no distinction of time behaviours, that both time variables are identical $t = \tau$ then we arrive to the implication of Lorentz's invariance for $\mathbf{A}_0$ and φ_0. Without additional hypothesis, the present Helmholtzian approach cannot rule in favour of Galilean or Lorentz's transformations for implicit time dependences. The novel gauge (90) as well as (92) is compatible with both of them. The only way of resolving this dilemma now seems to be to suggest experimental verification of electric field transformation in a moving frame. In fact, Leus recently proposed such experiment [16]. A uniform beam of electrons moving with the velocity close to c has to produce electric field strength which differs for Galilean and Lorentz transformations.

Two gauge conditions (90) and (91) can be written jointly in a more compact formula that we can call *the generalized Lorentz condition.*

$$\nabla\cdot\mathbf{A} + \frac{1}{c}\frac{d\varphi}{dt} = 0, \tag{93}$$

Where $\mathbf{A}_0$ and φ_0 are defined by (81) and the total time derivative is taken as in (69) up to the convection term. We have not done it yet and will do it elsewhere but, perhaps, it is possible to prove that generalized Lorentz gauge (93) is valid also for non-inertial frames (acceleration and higher order terms in the total time derivative expansion). It would have a very attractive consequence that the field equations (65)-(68) written in total time derivatives could be considered invariant regardless a frame of reference (inertial or non-inertial). Recall that in special relativity, electric and magnetic potentials of uniformly moving charge $\mathbf{A}_0$ and φ_0 are interrelated through the relationship (92) under application of Lorentz transformation. Here we found that relativistic potentials (or components of potential four-vector) are connected in a more general way (90). Another important aspect of the present approach can be attributed to the verification of some ambiguity in the use of Lorentz gauge since it is applicable only to explicit time-dependent potentials. In fact, there are some difficulties in the conventional electrodynamics concerning the inconsistency of this gauge with implicit time-dependent functions. The standards Lorentz gauge condition is assumed to be valid for total electric and magnetic potentials (transverse plus longitudinal) and is considered suffice to hold Maxwell's equations invariant under Lorentz transformation. In the quasi-stationary approximation, the Lorentz condition in every frame of references takes the form of the so-called radiation gauge [17].

$$\nabla\cdot\mathbf{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0 \tag{94}$$

$$\nabla\cdot\mathbf{A} = 0. \tag{95}$$

It contradicts the expected relation (92) (or in our approach (90)) between electric and magnetic implicit time-dependent potentials. To make (95) consistent with (92) in the given frame, they used to put an additional condition on the electric potential satisfying the so-called Coulomb gauge [17].

$$\nabla \cdot \mathbf{A} = 0 , \quad \varphi = 0 . \tag{96}$$

In mathematical language the invariance of implicit time-dependent fields in the conventional approach involves more strong limitations than those imposed previously by the Lorentz gauge. Generally speaking, the conventional classical electrodynamics has to admit more than one invariance principle since every time the Lorentz transformation is done, one needs also simultaneously to transform all physical quantities in accordance with the Coulomb gauge (96). This problem was widely discussed and in the language adopted in the general Lorentz group theory, is known as gauge dependent representation (or joint representation) of the Lorentz group [17]. In fact, it means an additional non-relativistic adjustment of electric potential, every time we change the frame of reference. This difficulty vanishes when the relativistic gauge (90) for implicit time-dependent potentials is introduced. A rigorous consideration of (84), (85) gives another important conclusion: simultaneous application of two independent gauge transformations (90), (91) discomposes the initial set (71)-(74) into two pairs of differential equations, namely.

$$\Delta \mathbf{A}_0 = -\frac{4\pi}{c} \varrho \mathbf{v} , \tag{97}$$

$$\Delta \varphi_0 = -\pi \varrho \tag{98}$$

At the same time with the homogeneous wave equations,

$$\Delta \mathbf{A}^* - \frac{4\pi}{c} \frac{\partial^2 \mathbf{A}^*}{\partial t^2} = 0, \tag{99}$$

$$\Delta \varphi^* - \frac{4\pi}{c} \frac{\partial^2 \varphi^*}{\partial t^2} = 0 . \tag{100}$$

Likewise (92), Poisson's second order differential equations (97), (98) for electric and magnetic potentials covers the conventional approach in the steady-state approximation and can be considered as valid extension to implicit time-dependent potentials. A general solution, as one would expect, satisfies a pair of uncoupled inhomogeneous D'Alembert's equations. It can be verified by summing up (97), (98) and (99), (100) (here we omit premeditatedly all boundary conditions for the sake of simplicity).

$$\Delta \mathbf{A} - \frac{4\pi}{c} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c} \varrho \mathbf{v} , \tag{101}$$

$$\Delta \varphi - \frac{4\pi}{c} \frac{\partial^2 \varphi}{\partial t^2} = -4\pi \varrho , \tag{102}$$

Where the total values $\mathbf{A}$ and φ are defined by (81). The same result has been derived in the Section 2 independently, starting from the analysis of boundary conditions for inhomogeneous D'Alembert's equations [18]. It has been shown mathematically that any general solution of Maxwell's equations has to be obligatory written as a superposition of implicit and explicit time-dependent functions. The above analysis endorsed that conclusion by demonstrating relativistic invariance of (101), (102) and, therefore, (71)-(74), if and only if the relativistic gauge condition (96) is satisfied by respective components of the total field. Thus, the covering theory based on the total time derivatives possesses all necessary relativistic symmetry properties. To conclude this section, some remarks worth to be done concerning the empirical and axiomatic status of the Lorentz force concept in the electron theory formulated by Lorentz. In the first version of Maxwell's theory published under the name "*On Physical Lines of Force*" (1861-1862) there was already admitted an unified character of a full electromotive force in the conductor in motion by describing it as [19,20].

$$\mathbf{E} = -\nabla \varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \frac{1}{c} \mathbf{v} \times \mathbf{H} , \tag{103}$$

Where (1) the first term is the electrostatic force, (2) the second one is the force of magnetic induction and (3) the third one is the force of electromagnetic induction due to the conductor motion. Later investigations began to distinguish between the electric force in a moving body and the electric force in the ether through which the body was moving and as a result, did not consider $\frac{1}{c} \mathbf{v} \times \mathbf{H}$ as a full-value part of the electric field, as afterwards was argued by Hertz. This distinction was one of the basic premises in Lorentz's electron theory and was closely related to the special status of the Lorentz force conception. It also can be noted in the way how it forms part the formalism of the conventional field theory . The equation of motion with total time derivative (34) should be contrasted from the form of partial differential equations (30)-(33). It does not correspond to the mathematical structure of a consistent system.

In special relativity the Lorentz force, is the result of the transformation of the components of Minkowski's force. Thus, the expression for the Lorentz force can be obtained in a purely mathematical way from the general relativistic relationships [2]. In the present Helmholtz-type approach the Lorentz force is one of the terms in the total time derivative expansion. This has advantage to be consistent by itself with the set of generalized field equations. There is no need to supplement Maxwell's theory with equation of motion. Given such interpretation of Lorentz's force, we remind that in our approach it can be related only to implicit time-dependent components whereas in the conventional electrodynamics it was the product of the total magnetic field leading to some ambiguities. In this respect it is interesting to mention very recent works by Wesley [21] and Phipps [22] challenging the sufficiency of the Lorentz force law to describe experimental observations. They advocated the use of total time derivatives (in the above-mentioned neo-Hertzian sense) and their data roughly agreed with theoretical predictions, while the conventional theory does not predict any effect at all.

Analysis of classical difficulties and the Hamiltonian form of generalized Maxwell's equations

Maxwell's equations in the form of D`Alembert's equations lends them to the covariant description and are in agreement with the requirements of special relativity mathematical formalism. For four-vectors of separated potentials, the standard four vector form of basic equations can be used. We immediately have the following expressions:

$$\left(\Delta-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\left(\mathbf{A}_{0\mu}+\mathbf{A}^*_{\mu}\right)=-\frac{4\pi}{c}\mathbf{j}_{\mu},\quad \mu=0,1,2,3\,, \tag{104}$$

where

$$\mathbf{A}_{0\mu}+\mathbf{A}^*_{\mu}=\left(\varphi_0+\varphi^*,\mathbf{A}_0+\mathbf{A}^*\right),\qquad \mathbf{j}_{\mu}=\left(c\varrho,\mathbf{j}\right). \tag{105}$$

The first Poisson's operator Δ acts only on the four-vector of implicit time-dependent components $\mathbf{A}_{0\mu}$ whereas Δ and $\partial^2/\partial t^2$ act together on explicit time-dependent components $\mathbf{A}^*_{\mu}$. The equation (104) is relativistically invariant under the generalized relativistic Lorentz gauge condition (93). T o give some substance to the above formality we exhibit explicitly Poisson's equation for implicit time-dependent four-vector $\mathbf{A}_{0\mu}$.

$$\Delta\mathbf{A}_{0\mu}=-\frac{4\pi}{c}\mathbf{j}_{\mu}\,, \tag{106}$$

where

$$\mathbf{A}_{0\mu}=\left(\varphi_0,\mathbf{A}_0\right). \tag{107}$$

As we demonstrated in the previous Section, equation (106) is relativistically invariant under the Lorentz gauge (86) if the time parameter τ here is considered identical to the time variable τ for explicit time components $\mathbf{A}^*_{\mu}$. Under this condition, Poisson's differential operator Δ acting on implicit time-dependent potentials becomes invariant in every inertial frame of reference under Lorentz's transformations. This is due to the fact that time variable t is not any more independent from τ as it is assumed for partial derivatives in full time derivative formalism. Non-covariant representation of D'Alembert differential operator $\Delta-\partial^2/\partial t^2$ or, in other words, non-covariance of equation (106) is not a stumbling block here for relativistic invariance and endorses the well-known fact that covariance is not necessary, it is only sufficient for relativistic invariance.

Moreover, it is tacitly implied in the conventional approach and corresponds to the relativistic invariance of field components of an uniformly moving charge (implicit time-dependent functions) that remain radial lines of electric field regardless the choice of inertial frame. This fact is odd to contemplate in the Faraday-Maxwell electrodynamics based on the concept of local (contact) field which mathematically fits explicit time- dependent behavior.

Actually, electric field lines of an unmoving charge are radial. Under Lorentz's transformation into the inertial frame of reference moving with the velocity $\mathbf{v}$ explicit time- dependence does not appear and field lines remain radial. Without any approximation, the influence of a possible retarded effect cancels itself at any distance from the moving charge. On the other hand, the conventional theory is unable to give any reasonable interpretation describing a transition from a uniform movement of a charge into an arbitrary one and then again into uniform over a limited interval of time. In this case, the first and the latter solutions can be given exactly by the Lorentz transformation as implicit time-dependent functions. What mechanism changes them at a distance unreachable for retarded Lienard-Wiechert fields? The lack of continuity between the corresponding solutions is obvious. It has the same nature as discussed in the Section 2.

The Helmholtz-type approach based on separation of implicit and explicit time behaviors also highlights serious ambiguities associated with the self-energy concept in the framework of the conventional electrodynamics. Let us confine our previous qualitative reasoning to the example of electrostatics. A rigorous analysis will be done later applying Hamiltonian formalism.

In electrostatics the total energy of N interacting charges is

$$W=\frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq 1}\frac{q_i q_j}{\left|\mathbf{r}_i-\mathbf{r}_j\right|}\,. \tag{108}$$

Here, the infinite self-energy terms $(i=j)$ are omitted in the double sum. The expression obtained by Maxwell for the energy in an electric field, expressed as a volume integral over the field, is [20]

$$W=\frac{1}{2}\int_V E^2 dV\,. \tag{109}$$

This corresponds to Maxwell's idea that the system energy must be stored somewhere in space. The expression (109) includes self-energy terms and in the case of point charges they make infinite contributions to the integral. In a relativistically covariant formulation the conservation of energy and the conservation of momentum are not independent principles. In particular, the local form of energy-momentum conservation can be written in a covariant form, using the energy- momentum tensor,

$$\frac{\partial T^{\mu\nu}}{\partial x^{\mu}}=0\,. \tag{110}$$

For an electromagnetic field, it is well-known that (110) can be strictly satisfied only for a free field (when a charge is not taken into account), whereas, for the total field of a charge this is not true, since (110) is not satisfied mathematically (four-dimensional analogy of Gauss's theorem). As everyone knows in classical electrodynamics, this fact gives rise to the "electromagnetic mass" concept, which violates the exact relativistic mass-energy relationship $\left(E=mc^2\right)$. Let us examine this problem in a less formal manner. The equivalent three-dimensional form of (110) is the formula (3). The amount of electrostatic self-energy of an unmoving charge in a given volume V is proportional to E^2 (see (109)). According to (110), in a new inertial frame, energy density w as well as electric field $\mathbf{E}$ must be, generally speaking, an explicit time-dependent function ($\partial w/\partial t\neq 0$ and $\partial\mathbf{E}/\partial t\neq 0$). On the other hand, the electric field strength of an unmoving charge keeps its implicit time behaviour under Lorentz's transformation $(\partial\mathbf{E}/\partial t=0)$. It contradicts the commonly accepted view that electrostatic self-energy is stored locally in space. In the framework of Helmholtzian approach these ambiguities can be cleared up. Actually, looking back at the general solution (27) with explicitly exposed longitudinal and transverse components, the term $\mathbf{E}_0$ is responsible for bipartite interaction between charges. No local energy conservation law in the form (110) or (3) is adequate for implicit time-dependent field $\mathbf{E}_0$. We suggest that the original mathematical form (108) should be used. Nevertheless, the local form (110) or (3) is perfectly adequate for explicitly time-dependent free field $\mathbf{E}^*$. Clear separation on implicit and explicit time dependencies in Helmholtz-type electrodynamics leads to the correspondent separation in the total electric field energy expression,

$$W = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq 1}\frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|} + \frac{1}{2}\int_V E^2 dV \,. \qquad (112)$$

This is a logical conclusion of our qualitative reasoning that will be mathematically verified below in Hamiltonian formulation. Let us discuss generalized field equations in total time derivatives (65)-(68) for arbitrary fields from the standpoint of the principle of least action. Applying explicitly separation of field components we have not done any modifications in the general four-vector representation of Maxwell equations (104), (105). We only noted that in this case the set of field equations can be split up for equations of implicit and explicit time-dependent potentials such as (20)-(23) or (97)-(100). A relativistic action for implicit time potential $A_{0\mu}$ can be written in the conventional form [2].

$$S_m + S_{mf} = \int_1^2 \left(-\sum_{a=1}^{N} m_a c ds_a - \sum_{a=1}^{N}\frac{q_a}{c}\sum_{\mu=0}^{3} A_0(ma) dx_a^{\mu} \right). \qquad (112)$$

This expression is sufficient to derive the first couple of equations (20), (21) (or (97), (98)) from the least action principle. It can be directly verified by rewriting the second term in (112) as

$$S_{mf} = -\frac{1}{c}\int \sum_{\mu} A_{0\mu} j^{\mu} dV \qquad (113)$$

And using Dirac's expression for four-current,

$$j_{\mu}(\mathbf{r},t) = \sum_a \left[-\frac{q_a}{4\pi}\Delta\left(\frac{a}{|\mathbf{r}-\mathbf{r}_a|}\right)\right]\mathbf{U}_{\mu a} \,, \qquad (114)$$

Where $\mathbf{U}_{\mu a}$ is the four-velocity of the charged particle a, and $\mathbf{r}_a$ is the radius vector of the particle a.

Let us consider the second pair of equations (22), (23) or (99), (100) defining explicitly time-dependent potentials $(\varphi^*, \mathbf{A}^*)$ or $\mathbf{A}^*_{\mu}$ in representation (104). It is easy to see that the conventional Hamiltonian form can be adopted to describe transverse components of electromagnetic field [2],

$$S_f = -\frac{1}{16\pi}\int \sum_{\mu,\nu} F_{\mu\nu} F^{\mu\nu} dV dt \,, \qquad (115)$$

Where

$$F_{\mu\nu} = \frac{\partial A_{\nu}^*}{\partial x^{\mu}} - \frac{\partial A_{\mu}^*}{\partial x^{\nu}} \,. \qquad (116)$$

Finally, it remains to be proved that the variational derivative,

$$\delta S_f = -\int \sum_{\mu}\left(\frac{1}{4\pi}\sum_{\nu}\frac{\partial F_{\mu\nu}}{\partial x^{\nu}}\right)\delta A_{\mu}^* dV dt \qquad (117)$$

Can be used to obtain the covariant analogue of (22), (23) or (99), (100) in the following form:

$$\sum_{\nu}\frac{\partial F^{\mu\nu}}{\partial x^{\nu}} = \sum_{\nu}\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{*\nu}}{\partial x_{\mu}} - \frac{\partial A^{*\mu}}{\partial x_{\nu}}\right) = 0 \,. \qquad (118)$$

The difference with the conventional interpretation consists in the way electromagnetic potentials $A_{0\mu}$ and A^*_{μ} take part in this Hamiltonian formulation. In the light of the Helmholtzian approach, the electromagnetic energy-momentum tensor demands some corrections in the interpretation of its mathematical formulation [2],

$$T^{\mu\nu} = -\frac{1}{4\pi}\sum_{\tilde{n}} F^{\mu}_{\tilde{n}} F^{\nu\tilde{n}} + \frac{1}{16\pi} g^{\mu\nu}\sum_{\beta,\gamma} F_{\beta\gamma} F^{\beta\gamma} \,. \qquad (119)$$

As a consequence of the definition (116), it can describe the energy-momentum conservation law for, exclusively, free electromagnetic field as follows.

$$\sum_{\nu}\frac{\partial T^{\mu\nu}}{\partial x^{\nu}} = 0 \,. \qquad (120)$$

Consequently, contrary to the traditional interpretation, the quantity $F^{\mu\nu}$ can be defined as a transverse electromagnetic field tensor because it contains only transverse field components but not total as in the conventional approach. There is no more violation of (120) even if the charge is taken into account, contrary to the situation in the conventional theory (see above discussion of equation (110)).

Strictly speaking, the total field energy W should be split up into two parts:

a. Energy W_{mf} of *longitudinal* implicit time-dependent fields responsible for electro- and loc magneto-static interaction between charges (*non-local* term) and

b. Energy W_f of *transverse* explicitly time-dependent electromagnetic field (*local* term),

$$W = W_{mf} + W_f \,. \qquad (121)$$

Following these results we suggest that the concept of potential (*non-local*) energy and potential forces must be re-established in classical electrodynamics. So, the system of charges and currents in absence of free electromagnetic field W_f must be considered as a conservative system without any idealization. Introduction of interaction energy W_{mf} in the form (112) equivalent to (108) definitely eliminates the problem of infinities of self-energy terms.

The physical meaning of the Pointing vector has been changed notably. So far the conventional theory dealt with it as a quantity describing dynamic properties of the total electromagnetic field. Now it is adequate only for conservation law in the form of equation (120) and, therefore, makes sense only for transverse components of electromagnetic field. Longtime well-known ambiguities related to the definition of the field energy location in space; do not take place in Helmholtz-type electrodynamics. In particular, there should be no flux of electromagnetic energy for stationary currents. Contrary, the conventional approach predicts senseless flux of energy coming from infinity towards the current [23].

At the end of this Section we would like to present a valuable mechanical analogy of Maxwell's equations in the form of (20)-

(23) or (97)-(100). It helps to understand why general solutions must be split up into (orthogonal) potentials (22), (23) (or (81)) with explicit and implicit time-dependence, respectively. The set of differential equations for elastic waves in an isotropic media [24] can be considered as mechanical analogy of Maxwell's equations to endorse Helmholtzian foundations of classical electrodynamics

$$\frac{\partial^2 \mathbf{u}_l}{\partial t^2} - c_l^2 \Delta \mathbf{u}_l = 0\ , \qquad (122)$$

$$\frac{\partial^2 \mathbf{u}_t}{\partial t^2} - c_t^2 \Delta \mathbf{u}_t = 0\ , \qquad (123)$$

Here c_l and c_t are spreading velocities of longitudinal and transverse waves, respectively.

The general solution of (113), (114) is the sum of two independent and orthogonal terms corresponding to longitudinal $\mathbf{u}_l$ and transversal $\mathbf{u}_t$ waves,

$$\mathbf{u} = \mathbf{u}_l + \mathbf{u}_t\ . \qquad (124)$$

If the longitudinal spreading velocity approaches formally to infinity ($c_l \to \infty$) then (122) transforms into Laplace's equation whereas the general solution turns out to have implicit time dependence. Solution (124) takes the form of separated potential solution (22), (23) (or (81)). Longitudinal component does not vanish in this limit from mathematical consideration, though the time behavior undergoes a fundamental transformation. Thus, longitudinal waves $\mathbf{u}_l$ have to be considered as full-value solution of the total system of differential equations (122), (123). It allows understanding why Hertz had no right to eliminate longitudinal components from mathematical solutions of Helmholtz's theory in Maxwellian limit (see Hertz's own words [25]: "...Helmholtz distinguishes between two forms of electrical force the electromagnetic and the electrostatic to which, until the contrary is proved by experience, two different velocities are attributed. An interpretation of the experiments from this point of view could certainly not be incorrect, but it might perhaps be unnecessarily complicated. In a special limiting case Helmholtz's theory becomes considerably simplified, and its equations in this case become the same as those of Maxwell's theory; only one form of the force remains, and this is propagated with the velocity of light. I had to try whether the experiments would not agree with these much simpler assumptions of Maxwell's theory. The attempt was successful. The result of the calculation is given in the paper on ' The Forces of Electric Oscillations, treated according to Maxwell's Theory'." [25].

To end this Section we conclude that the idea of non-local interactions is enclosed into the framework of Helmholtzian electromagnetic theory as unambiguous mathematical feature. On the other hand, some of the quantum mechanical effects like Aharonov-Bohm effect, violation of the Bell's inequalities etc. point out indirectly on the possibility of non-local interactions in electromagnetism. During the last century modern physics had faced fundamental difficulties in unifying relativistic classical physics elaborated mainly in the framework of the locality concept of relativistic theory and quantum physics characterized essentially by the emergence of non-locality. Regretfully, nowadays there is no rigorous mutual correspondence between these two fundamental areas of physical science. Helmholtz-type approach offers an altogether more promising solution.

Instead of conclusion

Almost all the above arguments we have taken verbatim from the previous work [6] of one of the authors of the present article. Did we resolve the problem of the intra-dipole radiation? No, we did not, may be... But as one can see, the problem of propagation of electromagnetic interactions cannot be considered as fully resolved by conventional classical electrodynamics. And we can see from previous sections that taking into account the double dependence (*implicit* and *explicit)* electrodynamics functions on time cannot help us to resolve the specific particular problem of the intra-dipole radiation…

However, the problem of the intra-dipole radiation could be resolved if we declare that only electric dipole must radiate electromagnetic waves rather than an electric charge. Indeed, how to theoretical physics got the idea that the accelerated charge must radiate? The vast majority of textbooks and monographs on classical electrodynamics are beginning to consider the process of emission of electromagnetic waves, starting with the study of the behavior of the *electric dipole*. Then, obtaining the formula for the total radiation of the *dipole*, they ignore the fixed dipole charge, usually located at the origin, and apply the mentioned formula to the moving second charge of the dipole. As an example, consider the textbook of Landau [2]: Unlike other books in [2] more accurately states that the charges *can* radiate only if they move with acceleration rather than *must*! Landau finds for the total radiation of the dipole

$$I = \frac{2}{3c^3}\ddot{\mathbf{d}}^2. \qquad (125)$$

Then he writes [2]: "If we have just one charge moving in the external field, then $\boldsymbol{d} = e\mathbf{r}$ and $\ddot{\boldsymbol{d}} = e\boldsymbol{w}$, where $\boldsymbol{w}$ is the acceleration of the charge. Thus (Landau writes) the total radiation of the moving charge *is*

$$I = \frac{2e^2w^2}{3c^3}." \qquad (126)$$

It is here that is hidden the deep logical error! The point is that $\boldsymbol{w} = \ddot{\mathbf{r}}$ in the beginning is the acceleration of the change of the vector $\mathbf{r}$ of the *intra-dipole distance* rather than an acceleration of the moving charge. Of course, if one of the dipole charges is at rest in this case $\mathbf{w}$ is the acceleration of the moving charge. But Landau [2] uses the following definition of the dipole moment of the system of charges

$$\mathbf{d} = \sum e_a \mathbf{r}_a\ , \qquad (127)$$

Where the origin is *anywhere* within the system of charges (it means that also in a point where is no any charge), and the radius vectors of the various charges are $\mathbf{r}_a$. Then Landau defines the dipole moment of *two* charges (positive and negative)

$$\mathbf{d} = e\mathbf{R}_{+-}\ , \qquad (128)$$

Where $\mathbf{R}_{+-}$ is the radius vector from the center of negative to the center of positive charge. Let us now return to the logical error mentioned above. Obtaining Equation (125) Landau [2] evaluates the amount of energy radiated by *the system of charges* in unit time into the element of solid angle do.

$$dI = \frac{1}{4\pi c^3}\left(\ddot{d}\times\mathbf{n}\right)^2 do = \frac{\ddot{\mathbf{d}}^2}{4\pi c^3}\sin^2\theta\, do. \qquad (129)$$

The point is that the intensity of radiation Equation (129) is obtained for *the complex* of charges *(for the dipole* $\mathbf{R}_{+-}$ in our case) rather than for a unit charge! However, the question arises: why, then, it is generally assumed that just an accelerated charge radiates electromagnetic energy (electromagnetic waves), rather than the dipole considered as an aggregate? In connection with the above, we believe that *an electric dipole is the most fundamental concept of electromagnetism with respect to electromagnetic radiation than a single electric charge.* It should be assumed also that not all of the time-varying dipoles emit, but only those in which the *modulus* of the dipole moment varies with time with acceleration that is clear from Equation (125). It could mean, for example, that the widespread belief that the classical hydrogen atom in which the electron moves in a *circular* orbit with a *constant* radius must radiate is wrong!

However the problem of the double dependence (*implicit* and *explicit)* electrodynamics functions on time remains open and requires further research. The logical analysis of Maxwell-Lorentz equations for one charge system shows ambiguous conventional treatment of *implicit* and *explicit* time dependencies. It was found that all conventional approach is beset with the same ambiguities leading to many mathematical inconsistencies and paradoxes.

We suggested that it is possible to solve those difficulties by clear distinguishing between functions with *implicit* and *explicit* time dependencies. This consideration provided self-consistency for mathematical description of electromagnetic theory. Maxwell's equations resulted to be written in full time derivatives that consistently covers conventional approach. We showed that the covering theory possesses all necessary relativistic invariance properties for inertial frames of references. Usual Lorentz's gauge condition is covered by generalized gauge condition. It promises to keep generalized Maxwell's equations invariant also in non-inertial frames but this issue will be studied elsewhere.

Consistent mathematical interpretation of generalized field equations gives a solid ground for Helmholtzian foundations of classical electrodynamics [25,26] based on the superposition of *implicit* time-dependent *longitudinal* and *explicit* time-dependent *transverse* components. This approach demonstrates advantages over the conventional field description in eliminating the large number of internal inconsistencies from classical electrodynamics and promises more adequate solution to fundamental problems of modern physics. Recent experimental data [21,22] highlighted certain limitations of the conventional approach. Graneau's monograph on modern Newtonian electrodynamics [27] reviewed numerous research data in exploding wires, railguns, different electromagnetic accelerators, jet propulsion in liquid metals, arc plasma explosions, capillary fusion etc. as unambiguous indication on the existence of non-local *longitudinal forces*. Thus, a new area of electromagnetic research emerges that is interested in the study of *longitudinal* components by experimental as well as by theoretical means.

We would like to hope that our article will attract researchers' interest to the unresolved problems of classical electrodynamics, which, *remaining unresolved*, directly *migrated* to *quantum* mechanics and electrodynamics!

As a final conclusion of this paper, we would like to quote Duhem's significant words [28]: " ... An excessive admiration for Maxwell's work has led many physicists to the view that it does not matter whether a theory is logic al or absurd, all it is required to do is suggest experiments: A day will come, I am certain, when it will be recognized: that above all the objects of a theory is to bring classification and order into the chaos of facts shown by experience. Then it will be acknowledged that Helmholtz's electrodynamics is a fine work and that I did well to adhere to it. **Logic can be patient, for it is eternal**".

References

1. Arnoldus YF, Berg M J (2014) Energy transport in the near field of an electric dipole near a layer of material. Journal of Modern Optics 62(3).
2. Landau LD, Lifshitz EM (1985) Classical Theory of Field (3rd edn). Oxford: Pergamon press, UK, pp. 1-387.
3. Chubykalo A, Espinoza A, Tzonchev R (2004) Experimental test of the compatibility of the definitions of the electromagnetic energy density and the Poynting vector. The European Physical Journal D 31: 113-120.
4. Dirac PAM (1978) Directions in Physics. John Wiley & Sons, New York, USA.
5. Woodruff AE (1968) The Contribution of Hermann von Helmholtz to Electrodynamics. ISIS 59(3).
6. Chubykalo A, Smirnov-Rueda R (2001) Modern Helmholtzian Electrodynamics as a Covering Classical Electromagnetic. Theory Electromagnetic Phenomena 2: 1-28.
7. Tikhonov AN, Samarski AN (1963) Equations of Mathematical Physics (2nd edn). Oxford, Pergamon Press, UK.
8. Villecco RA (1993) Instantaneous action-at-a-distance representation of field theories. Physical Review E 48(5): 4008-4026.
9. Hylleraas EA (1970) Mathematical and Theoretical Physics Volume 2. (1st edn), Wiley-Interscience, New York, USA.
10. Hertz H (1890) Wied Annalen 41: 369
11. Hertz H (1894) Ges Werke. 2: 256.
12. Jackson LD (1963) Classical Electrodynamics. John Wiley & Sons, New York, USA, pp. 1-656.
13. Phipps TE jr (1986) Heretical Verities: Mathematical Themes in Physical Description (1st edn). Classical Non-fiction, Urbana, USA.
14. Phipps T E jr (1993) On Hertz's Invariant Form of Maxwell's Equations. Physics Essays 6(2): 1-15.
15. Chubykalo A, Smirnov-Rueda R (1997) Convection Displacement Current and Generalized Form of Maxwell-Lorentz Equations. Modern Physics Letters A 12(1).
16. Leus V (1999) Proceedings of the 2nd Siberian Conference on mathematical problems of physics. Institute of Mathematics (Novosibirsk: Russian Academy of Sciences), Russia, pp. 134-148.

17. Barut AO (1980) Electrodynamics and Classical Theory of Fields and Particles. Dover Publications. Inc, USA.

18. Chubykalo A, Smirnov-Rueda R (1996) Action at a Distance as a Full-value Solution of Maxwell equations: The basis and Application of the Separated-Potentials Method. Physical Review E 53(5): 5373-5381.

19. Maxwell JC (1864) On Physical Lines of Force. Scientist Papers.

20. Maxwell JC (1892) Treatise on Electricity and Magnetism. Oxford Univ. Press, London.

21. Wesley JP (1998) The Marinov Motor, Notional Induction without a Magnetic B Field. Apeiron, p.1-7.

22. Phipps TE jr (1998) Observations of the Marinov Motor. Apeiron, p. 1-16.

23. Feynman R P (1964) Lectures on Physics: Mainly Electromagnetism and Matter. Addison-Wesley, New York, USA.

24. Landau LD, Lifshitz EM (1970) Theory of Elasticity. Oxford: Pergamon press, UK.

25. Hertz H (1962) Electric Waves. Dover Publications. Inc, USA.

26. Helmholtz H (1882) Wissenschaftliche Abhandlungen. Barth, Germany.

27. Graneau P, Graneau N (1996) Newtonian Electrodynamics. World Scientic Publish. Inc, Singapore.

28. Duhem P (1905) Les Theories Electriques de J. Clerk Maxwell (Paris).