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On the Existence of Gauge Functions for Space-Like Separations

Andrew Chubykalo[1], Augusto Espinoza[1], Rolando Alvarado-Flores[2]

[1]*Unidad Academica de Fisica, Universidad Autónoma de Zacatecas, A.P. C-580, Zacatecas, Mexico*
[2]*Centro de Estudios Multidisciplinarios, Universidad Autónoma de Zacatecas, Av. Insurgentes 108A, tercer piso, col. Centro, Zacatecas, Zac, México. C.P. 98000*

***Corresponding Author: Andrew Chubykalo,** *Unidad Academica de Fisica, Universidad Autónoma de Zacatecas, A.P. C-580, Zacatecas, Mexico*

Abstract: *In this paper we have a threefold objective. 1. To propose a method for the determination of gauge transformations. 2. to prove that some gauge transformations are local, in the sense that they are not defined all along the space-time. 3. To show that the Cauchy problem for Maxwell equations depends on the geometry of space-time, so the choice of gauge cannot be convential. After developing our method we compare it with Jackson´s well known procedure of [1]. Our first result is that the methods are not coextensive, but they have, however, a common class of solutions in some important cases, e.g. the class of gauge functions transforming the Lorenz gauge into the velocity gauge which is treated in detail. However, our scope in the paper is wider and much more theoretical than that of [1], because we advance to prove that there are certain regions in space-time where no gauge transformation exists, so the results of any method to determine a gauge function are "local" in the sense that they produce results valid for a bounded space-time región, and not for all. So our second and main result in the paper is that gauge transformations in electromagnetic theory are always local. We give also a discussion of the Cauchy problem for classical space-times in order to show that Maxwell equations for these space-times allow instantaneous solutions.*

Keywords: *Gauge transformations, space-like surfaces*

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1. Introduction

We want to start with the following quotation from [1] (p. 917):

"…the textbooks rarely show explicitly the gauge function f that transforms one gauge into another".

So one of Jackson´s aims in [1] is to show a whole class of gauge functions for several gauge conditions. The other aim was to prove that whatever the gauge conditions imposed on the Maxwell´s equations for the potentials the solutions are always causal and propagating with the speed of light c. Hence, according to this standpoint, within the framework of electromagnetic theory no non-causal solution propagating faster than light speed is allowed. We don´t share the hopes behind this position. On the contrary, Maxwell´s equations accepts non-causal and propagating faster than light solutions. However we think that Maxwell´s equations plus the special theory of relativity does not allow such solutions. Even more, if this is the case, the right gauge condition that restricts the solutions of Maxwell´s equations to this class is the Lorenz gauge because it is a Lorentz´s invariant gauge. There are several entangled theoretical positions here, let us try to disentagle them all for clarity.

1. We identify the theorethical framework of electromagnetic theory with Maxwell´s equations. Hence, within this framework causal and propagating at the speed of light solutions are those solutions symetrical under the action of the Lorentz group. If the Maxwell´s equations admit any other group their solutions will share that symmetry. That´s our point of view about the space of solutions of Maxwell´s equations. So, it is evident that if we choose an absolut framework and pick up the solutions invariant under the euclidean group (space-time translations plus spatial rotations) we obtain instantaneous solutions, which are obvious faster than the speed of ligth solutions. Of course, the general consensous in the scientific community is that no such framework (the ether) exists, because of the negative results of the Michelson-Morley experiment, so this reading of Maxwell´s equations is not experimentally supported.

2. When the people try to solve Maxwell´s equations by means of potentials sooner or later discovers that there are many potentials allowed just by specifying "gauge conditions", and that all of them are related by means of gauge transformations. Jackson´s describes this "tortuous path" in [2]. The set of potentials defined by particular gauge conditions are equivalent to another set in another gauge in the sense that all produce the same electromagnetic field. If this is the case the idea that the choice of gauge conditions is conventional, in the sense that, due to the existence of an invertible gauge transformation among different potentials it is possible that any of them can be picked up by "convention", is quite natural. We dont believe in this theoretical position: the gauge conditions, we stand, are related to the geometry of space-time in the sense that only a particular gauge respects the space-time symmetries, and this is the right gauge for that space-time. Indeed, what we must do is to give theoretical reasons to choose a gauge not respecting .the space-time symmetries. Even more, we are going to prove in this paper that for some cases no gauge function exists to transform some given gauge into another. This gives, we think, plausibility to our interpretation.

3. In the long review by Recami [3] massive argumentation and bibliography is offered around the possibility of faster than the speed of light objects within the framework of the Special Theory of Relativity (STR). As we have said we dont believe in faster than the speed of light objects within the framework of Maxwell´s equations and the STR. In [3] this problem is considered until part V.15. Our position here is that in the kinematics of special relativity any faster than ligth object is not possible, however is not a contradictory option. But we have no good arguments beyond the standard ones. However, when we consider STR plus Maxwell´s equations we have more clear ideas. Looking at Recami´s treatment to generalize Maxwell´s equations to include tachyons our conclusión is that this generalization is just like Bilaniuk, Desphande, Sudarshan proposal in [4], or in Feinberg [5] of an imaginary mass. The comments by Robinett in [6] we believe conclusive: the global hiperbolicity of Klein-Gordon and D´Alembert equations forbid any faster than light solution. However faster than light objects are posible if the global geometry of space-time turns out to be euclidean. This is an empirical question. Obviously we reject the "metaphysics" of Recami in [3](p.13) which says:

"…everything that was thinkable without meeting contradictions exists somewhere in the unlimited universe"

This is clearly flawed, as shows the non euclidean geometries: something may be consistent, but from it we cannot deduce that it exists Indeed this is a variant of the ontological argument to prove God existence. If faster than the speed of light objects do exists at all, they must be detected experimentally.

In this paper we want to propose a method for the determination of gauge functions and to show that no global gauge transformations exists for some cases. This is going to be the theorethical support for our idea that the potentials are not conventional, and for our interpretation that Maxwell´s equations, being independent of space-time geometry because of its general covariance, become geometry dependent once we choose a gauge. Hence we requiere an exact understanding of the local/global distinction. Let´s consider a smooth d −dimensional manifold M. Our mathematical representation of this differentiable object is that it is the set theoretical union of intersected sets $U_1, \dots, U_K$ such that:

1. $M = \cup_{i=1}^{K} U_i$
2. $U_i \cap U_j \neq \emptyset \;\; i \neq j$

With each set U_i we attach a smooth map $\Delta_i : U_i \to R^d$ with R^d the d-dimensional real space, to form a *local chart* $\mathbf{U}_i = < \Delta_i, U_i >$ which is an ordered pair. The smooth maps satisfy:

3. $g_{ij} = \Delta_i^{-1} \circ \Delta_j : U_i \cap U_j \to U_i \cap U_j$
4. $g_{ij}^{-1} = \Delta_j \circ \Delta_i^{-1} : O \to O$ where $O \subset R^d$

The functions g_{ij} are usually called *"transition functions"*. Whenever we can establish a convention or property in all the charts we say we have a *"global convention"* or a *"global property"*, otherwise our conventions or properties are local. In [7] Kiskis showed how in a non-simply connected manifold charge conservation is not a global property. What is important for us is the following comment: *"The topology of space-time on a cosmological scale is not known"*. We believe that topology is still unknown. However we have a piece of information: there is a limit for the velocity of field propagation. With this information we can introduce physical content in the local/global distinction. A physical property is "local" if an only if it is time-like, i.e. it is within the future directed or past directed light cone. It is "global" if it is hypothetically distributed all along space-time. An instantaneous solution is global in this sense, but a solution propagating at the speed of light is local except when $t \to \pm\infty$. This paper is organized as follows. In section **II** we introduce the velocity and Lorenz gauges within the context and conventions of [8] because we want to discuss their theory of faster than light objects. In **III** we introduce our method to obtain gauge functions. In **IV** we show that the physical speculations of Brown and Crothers (BC) cannot stand the way they are stated. This paper is important on their own because, according to Yang [9] in this paper the mathematical groundwork for the velocity gauge was developed. More important for us are some solutions obtained by BC. In **V** we prove that for some cases Jackson´s method for obtaining gauge functions coincides with ours, but in general they are not coextensive. Section **VI** is devoted to the discussion of the Cauchy problem for Maxwell´s equations in spaces of simultaneity, proving that for this case the Cauchy data are continuous all along the spaces of simultaneity, but this case is not compatible with the special theory of relativity. In section **VII** we propose a proof of locality of gauge transformations.

2. PRELIMINARY DEFINITIONS

In the paper [8] we find a good discussion of the relations between the α-Lorenz gauge, introduced by Yang in [10], see also [9] for more information, and the Lorenz gauge. For the sake of simplicity we shall call the α-Lorenz gauge "v-gauge". Jackson in [1] calls this gauge "velocity gauge". We are interested in two results of the paper by Brown and Crothers (BC):

1. Its claim in page 2950 that "…there is no gauge transformation of the second kind which transforms between the Lorentz and the α-Lorentz gauge".

2. Their interpretation of the solution to the Brown-Crothers equation given in section 4, i.e. we are going to prove that their equations (65a-b) are just another way to express the gauge conditions, hence, what they are trying to do is to give physical meaning to the gauge conditions themselves.

The name "Brown-Crothers equation" is taken from [9], and we shall adopt it. These two points contain ideas with which we feel identified, however we believe that they cannot stand in the form advanced by these authors. We discuss them in detail in part **IV** below. Because we need the field equations for the potentials in the Lorenz and v gauges it is useful to remember them both.

The field equations for the potentials in the Lorenz gauge (in gaussian units) are:

$$\mathbf{A}(\mathbf{x},t) = -\frac{4\pi}{c}\mathbf{J}\,, \tag{1}$$

$$\varphi = -4\pi\varrho\,, \tag{2}$$

$$\frac{1}{c}\frac{\partial\varphi}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{A} = 0\,, \tag{3}$$

Where $= \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ and ϱ and $\mathbf{J}$ are the usual matter fields (charge and current densities). While in the v-gauge, using the Brown-Crothers representation, they are:

$$\mathbf{A}_v(\mathbf{x},t) + \gamma\boldsymbol{\nabla}\big(\boldsymbol{\nabla}\cdot\mathbf{A}_v(\mathbf{x},t)\big) = -\frac{4\pi}{c}\mathbf{J} \tag{4}$$

$$_v\varphi_v = -4\pi\varrho\,, \tag{5}$$

$$\frac{1}{\alpha^2 c}\frac{\partial\varphi_v}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{A}_v = 0\,, \tag{6}$$

Where $\gamma = \alpha^2 - 1$ and $_v = \Delta - \frac{1}{\alpha^2c^2}\frac{\partial^2}{\partial t^2}$. Jackson in [1] follows another convention for the v-gauge that we shall not consider, here we follow Brown and Crothers [8] (p. 2943). There is another representation for the field equations in the v-gauge:

$$\alpha^2{}_v\,\mathbf{A}_v(\mathbf{x},t) + \gamma\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\mathbf{A}_v(\mathbf{x},t) = -\frac{4\pi}{c}\mathbf{J}\,, \tag{7}$$

$$_v\varphi_v = -4\pi\varrho\,. \tag{8}$$

In this representation the use of the Helmholtz theorem is quite easy, see [15].

3. GAUGE FUNCTIONS BY A NEW METHOD

In this section we are going to introduce our method to obtain gauge functions. We shall do it stating explicit propositions with proofs. We have the following

Proposition 1: The conditions

a. The gauge function f is of class S^4 (i.e. with continous partial derivatives up to the order 4, bounded at infinity)

b. The gauge function satisfies:

$$ {}_v f = \frac{4\pi\gamma}{\alpha^2 c}\frac{\partial\varrho}{\partial t} \tag{1}$$

b. There exist solutions for the field equations **II.** (1)- (2)-(3)-(4).

c. The leading operators , $\square_\alpha$ commute.

They are necessary and sufficient for the existence of the invertible gauge transformation:

$$\mathbf{A} = \mathbf{A}_v + \boldsymbol{\nabla} f \ , \tag{2}$$

$$\varphi = \varphi_v - \frac{1}{c}\frac{\partial f}{\partial t} \ , \tag{3}$$

i.e., transform solutions in the Lorenz gauge to the v-gauge.

Proof: The proposition includes a double implication: if the conditions (a-b-c-d) are satisfied, then there is a gauge transformation (2-3) hence they are sufficient. Now, if the gauge transformation (2-3) exists and is invertible, the conditions (a-b-c-d) are satisfied, so they are necessary.

Let us start proving the necessity, i.e., if the gauge transformation exists and is invertible, the gauge function satisifies conditions (a-b-c-d). To construct (2-3) we requiere solutions to the field equations, without these solutions we can do nothing, so condition (b) is necessary. To prove the necessity of conditions (c-d) we take the divergence in equation (2) and a partial time derivative of the form $\frac{1}{c}\frac{\partial}{\partial t}$ in equation (3). So, we use the gauge conditions to write equation (2) as follows:

$$\frac{1}{c}\frac{\partial\varphi}{\partial t} = \frac{1}{\alpha^2 c}\frac{\partial\varphi_v}{\partial t} - \Delta f$$

And equation (3) becomes:

$$\frac{1}{c}\frac{\partial\varphi}{\partial t} = \frac{1}{c}\frac{\partial\varphi_v}{\partial t} - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

So we get:

$$ f = -\frac{\gamma}{\alpha^2 c}\frac{\partial\varphi_v}{\partial t} \ . \tag{4}$$

If in equation (3) we use the operator $\frac{1}{\alpha^2 c}\frac{\partial}{\partial t}$ and proceed in the same way we obtain:

$$ {}_v f = -\frac{\gamma}{\alpha^2 c}\frac{\partial\varphi}{\partial t} . \tag{5}$$

So equations (4) and (5) must be satisfied by the gauge function f. Now we start using the gauge transformation from the v gauge to the Lorenz gauge

$$\mathbf{A}_v = \mathbf{A} + \boldsymbol{\nabla} g \ , \tag{6}$$

$$\varphi_v = \varphi - \frac{1}{c}\frac{\partial g}{\partial t} , \tag{7}$$

Following the same easy steps we can see that we now have for the gauge function g:

$$ g = \frac{\gamma}{\alpha^2 c}\frac{\partial\varphi_v}{\partial t} \ , \tag{8}$$

$$ {}_v g = \frac{\gamma}{\alpha^2 c}\frac{\partial\varphi}{\partial t} , \tag{9}$$

Now it is clear from equations (4-5), (8-9) that they share a common solution if: $f = -g$. If this is the case the gauge transformation is invertible. So the conditions for invertibility are the equations (4-5) only.

Now with the help of the conventions in the appendix, by parts integration and Green´s functions properties, we can write equations (4-5) in the following way [see equation (A8) in the appendix]:

$$
\begin{aligned}
{}_{v}f &= -\frac{\gamma}{\alpha^2 c}\frac{\partial}{\partial t}\left(G_\alpha(\mathbf{x},t;\mathbf{x}',t')\middle|\varrho(\mathbf{x}',t')\right) \\
&= -\frac{\gamma}{\alpha^2 c}\left(G_\alpha(\mathbf{x},t;\mathbf{x}',t')\middle|\frac{\partial}{\partial t'}\varrho(\mathbf{x}',t')\right),
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
f &= -\frac{\gamma}{\alpha^2 c}\frac{\partial}{\partial t}\left(G_L(\mathbf{x},t;\mathbf{x}',t')\middle|\varrho(\mathbf{x}',t')\right) \\
&= -\frac{\gamma}{\alpha^2 c}\left(G_L(\mathbf{x},t;\mathbf{x}',t')\middle|\frac{\partial}{\partial t'}\varrho(\mathbf{x}',t')\right).
\end{aligned} \tag{11}
$$

If we apply the operator to (10) and the operator ${}_{v}$ to (11), and again use Green´s functions properties, we obtain:

$$
{}_{v}f = \frac{4\pi\gamma}{\alpha^2 c}\frac{\partial}{\partial t}\varrho(\mathbf{x},t), \tag{12}
$$

$$
{}_{v}f = \frac{4\pi\gamma}{\alpha^2 c}\frac{\partial}{\partial t}\varrho(\mathbf{x},t). \tag{13}
$$

The equations are clearly the same if the operators commute, as they do in the functional space S^4. If we suppose that the operators doesn´t commute we obtain a contradiction using: ${}_{v}f - {}_{v}f = 0$. So, condition (c) is necessary. Then, the equation that must satisfy f is (7) because it is deduced form (c). So is necesary also. Hence if the gauge transformation exists and is invertible, the conditions (a-b-c-d) are satisfied. So the conditions are necessary conditions.

Now we shall prove sufficiency, i.e. if the gauge function exists and satisfies (4)-(5) the gauge transformation exists and is into the v-gauge. To do so we suppose the gauge transformation:

$$
\mathbf{A} = \mathbf{A}_e + \nabla f, \tag{14}
$$

$$
\varphi = \varphi_e - \frac{1}{c}\frac{\partial f}{\partial t}, \tag{15}
$$

Where the fields $\langle \mathbf{A}_e, \varphi_e \rangle$ are defined in an unknown gauge, but f satisfies (4)-(5). If this is the case the gauge transformation (14)-(15) must be into the v gauge. To see that this is indeed the case we take the divergence in (14) to obtain:

$$
-\frac{1}{c}\frac{\partial\varphi}{\partial t} = \nabla\cdot\mathbf{A}_e - \Delta f, \tag{16}
$$

But this becomes:

$$
{}_{v}f = \frac{\gamma}{\alpha^2}\nabla\cdot\mathbf{A}_e - \frac{\gamma}{\alpha^2}\Delta f. \tag{17}
$$

If we use (5). After some algebra we get:

$$
\frac{1}{\alpha^2}f = \frac{\gamma}{\alpha^2}\nabla\cdot\mathbf{A}_e. \tag{18}
$$

Now we use (4) to write:

$$
-\frac{\gamma}{\alpha^4 c}\frac{\partial\varphi_y}{\partial t} = \frac{\gamma}{\alpha^2}\nabla\cdot\mathbf{A}_e. \tag{19}
$$

We can see that (19) is the v gauge. So $\mathbf{A}_e = \mathbf{A}_v$. **QED.** This result is clearly local because it is defined pointwise, in a neigbourhood of a point where all derivatives make sense.

Proposition 2: The gauge function f transforming the Lorenz gauge into the v gauge is given by:

$$
f(\mathbf{x},t) = \frac{\gamma}{\alpha^2}\left(\Gamma(\mathbf{x},t;\,\mathbf{x}',t')\middle|\frac{\partial}{\partial t'}\varrho(\mathbf{x}',t')\right)
$$

With

$$
\Gamma(\mathbf{x},t;\,\mathbf{x}',t') = \frac{1}{c}\left(G_L(\mathbf{x},t;\mathbf{x}'',t'')\middle|G_\alpha(\mathbf{x}'',t'';\mathbf{x}',t')\right)
$$

Proof: Equation (1) is quite similar to equation (39) in [8]. Hence the solution is pretty much the same as (37) of [8], but we can obtain it by means of a simple procedure. We separate equation (1) with the help of the auxiliary function σ as follows:

$$f = \sigma \,, \tag{20}$$

$$_v\sigma = \frac{\gamma}{\alpha^2 c}\frac{\partial \varrho}{\partial t}\,. \tag{21}$$

The solution is now easy to obtain following the adequate boundary value problems (see the boundary value problem in [12]):

$$\sigma(\mathbf{x},t) = \frac{\gamma}{\alpha^2 c}\left(G_\alpha(\mathbf{x},t;\mathbf{x}',t')\middle|\frac{\partial}{\partial t'}\varrho(\mathbf{x}',t')\right), \tag{22}$$

$$f(\mathbf{x},t) = \left(G_L(\mathbf{x},t;\mathbf{x}',t')\middle|\sigma(\mathbf{x}',t')\right). \tag{23}$$

So

$$f(\mathbf{x},t) = \frac{\gamma}{\alpha^2 c}\left[G_L(\mathbf{x},t;\,\mathbf{x}'',t'')\middle|\left(G_\alpha(\mathbf{x}'',t'';\mathbf{x}',t')\middle|\frac{\partial}{\partial t'}\varrho(\mathbf{x}',t')\right)\right], \tag{24}$$

Or, after some algebra (and Fubini´s theorem):

$$f(\mathbf{x},t) = \frac{\gamma}{\alpha^2}\left(\Gamma(\mathbf{x},t;\,\mathbf{x}',t')\middle|\frac{\partial}{\partial t'}\varrho(\mathbf{x}',t')\right), \tag{25}$$

Where

$$\Gamma(\mathbf{x},t;\,\mathbf{x}',t') = \frac{1}{c}\left(G_L(\mathbf{x},t;\,\mathbf{x}'',t'')\middle|G_\alpha(\mathbf{x}'',t'';\,\mathbf{x}',t')\right). \tag{26}$$

QED

So, in this section we have developed our method to get gauge functions as solutions of a boundary value problem of fourth order. More information about these boundary value problems can be obtained in [8]

4. Discussion of Brown-Crothers results

Now we are going to discuss in detail the points 1-2 of section II up. In section 3 of [8] Brown and Crothers solve the field equations (4)-(5), and write the solution in the form (we use our notation):

$$\mathbf{A}_v = \mathbf{A} + \nabla g\,, \tag{1}$$

$$\varphi_v = \varphi - \frac{1}{c}\frac{\partial g}{\partial t}\,. \tag{2}$$

Certainly the solution looks like a gauge transformation from the v gauge to the Lorenz gauge, but for Brown and Crothers the piece g contains all the information related to the possibility of superluminal velocities ($\alpha > 1$). They indicate that g satisfy our equation (9) (or (63) in [1]), so they make the substitution of (1)-(2) in **II**.4 (or 27a in [1]) to obtain the following equation:

$$\square g + \gamma\nabla(\Delta g) = -\gamma\nabla(\nabla\cdot\mathbf{A})\,. \tag{3}$$

Here of course we use our notation. We want to prove the following:

Proposition 3: The equation (3) is equivalent to the Lorenz gauge condition .

Proof

$$\square g = {}_v\square g - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\square g + \frac{1}{\alpha^2 c^2}\frac{\partial^2}{\partial t^2}\square g = {}_v\square g - \frac{\gamma}{\alpha^2 c^2}\frac{\partial^2}{\partial t^2}\square g\,. \tag{4}$$

So equation (3) becomes:

$$\square_v g + \gamma\nabla\left(\Delta g - \frac{1}{\alpha^2 c^2}\frac{\partial^2}{\partial t^2}g\right) = -\gamma\nabla(\nabla\cdot\mathbf{A}) = \nabla(\alpha^2{}_v g)\,. \tag{5}$$

Hence we have the equation:

$$\gamma \nabla(\nabla \cdot \mathbf{A}) + \nabla({\alpha^2}_v g) = 0\,, \tag{6}$$

Using **III. (**9) we obtain:

$$\nabla\left(\nabla \cdot \mathbf{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t}\right) = 0. \tag{7}$$

QED

So the equation (3) is not a new equation at all, but the Lorenz gauge condition, hence equation (65a) in [8] is also nothing else but the gauge condition. The same is true about equation (65b) in [8] but the proof is similar so we may skip it.

We can see that our function $\Gamma(\mathbf{x}, t;\ \mathbf{x}', t')$ defined in **III.**(26) is the function $-G(\mathbf{x}\, t;\ \mathbf{x}', t')$ given by equation (37) in [1]. Hence we have an explicit expression in (38) of [1]. But we can see also that our gauge function (25) is quite exactly the function (62) in [1]. The conclusión is that to obtain a solution in the v gauge all we need is a solution in the Lorenz gauge plus the gauge function (25). Therefore we have obtained that solution using another method (Yang in [9] III-F also obtains the solution by his own means). But Brown and Crothers requiere the non-invertibility of the gauge transformation as support of their interpretation, because we can also obtain a solution in the Lorenz gauge solving the field equations for the potentials in the v gauge plus the gauge function; and this is the possibility allowed by proposition 1 above. Therefore if this is the case we can say that we have isolated the components moving at the velocity of light, leaving just the superluminal ones, so the electromagnetic field depends on α (see also [15]). Of course if we remember the accepted doctrine of the "conventionality" of the potentials (i.e. we choose among them selecting a gauge condition by arbitrary convention) we can say that no physical meaning can be given to any one of them, just the gauge invariant fields, like the electromagnetic field, are physically meaningful. So any tachyon field can be ruled out using a gauge transformation. The underlying physical idea is that no object can move faster than light and that any potential involving this possibility is just an artifact. Clearly, and we want to stress this point of view, if we suppose that indeed no object can move faster than light we shall not find any possibility for such an object using our mathematics because we are *a priori* restricting the set of possible solutions. We remark again: the existence of faster than the speed of light objects is an empirical question.

This could be the conclusión of this discussion: no tachyon field is allowed by means of gauge conditions like the v condition within the conventional interpretation of the potentials.

5. NON-EQUIVALENCE OF JACKSON'S METHOD AND OUR PROPOSED METHOD

In this section we shall prove that Jackson´s method for the calculation of the gauge function f in [1] (VII.A p. 923) from the Lorenz to the velocity gauge (which we call here v gauge) is not in conflict with the method for this calculation that we have developed in **III**, indeed, our method gives always a solution that satisfies Jackson´s conditions. So we want to prove that **III.**(26) is not in conflict with Jackson´s (7.5). To do so we define:

$$\tau(z) = t - t' - \frac{R}{z}$$

Choosing, as is usually the case, the lower half of the light cone defined by τ on space-time (see, e. g. [11] p. 189, see also the appendix). With $R = |\mathbf{x} - \mathbf{x}'|$ and $v = \alpha c$ we can write:

$$\breve{G}(\mathbf{x}, t;\ \mathbf{x}', t') = \frac{\delta\left(t - t' - \frac{R}{v}\right)}{R} - \frac{\delta\left(t - t' - \frac{R}{c}\right)}{R}\,. \tag{1}$$

Our first step is to obtain a differential equation for (1). Now, if we apply the operator □ to $\breve{G}(\mathbf{x}, t;\ \mathbf{x}', t')$ we obtain:

$$\breve{G}(\mathbf{x}, t;\ \mathbf{x}', t') = \left[\frac{\delta\left(t - t' - \frac{R}{v}\right)}{R}\right]$$

$$+4\pi\delta(\mathbf{x} - \mathbf{x}')\delta(t - t')\,.$$

But:

$$\left[\frac{\delta\left(t-t^{'}-\frac{R}{v}\right)}{R}\right] = {}_{v}\left[\frac{\delta\left(t-t^{'}-\frac{R}{v}\right)}{R}\right] + \frac{c^2-v^2}{(cv)^2}\frac{\partial^2}{\partial t^2}\frac{\delta\left(t-t^{'}-\frac{R}{v}\right)}{R}.$$

So

$$\breve{G}(\mathbf{x},t;\ \mathbf{x}^{'},t^{'}) = \frac{c^2-v^2}{(cv)^2}\frac{\partial^2}{\partial t^2}\frac{\delta\left(t-t^{'}-\frac{R}{v}\right)}{R}$$

But now we can see that:

$${}_{v}\breve{G}(\mathbf{x},t;\ \mathbf{x}^{'},t^{'}) = -4\pi\frac{c^2-v^2}{(cv)^2}\ \delta(\mathbf{x}-\mathbf{x}^{'})\frac{\partial^2}{\partial t^2}\delta(t-t^{'})\ . \qquad (2)$$

This equation is not explicit in the treatment of Brown and Crothers. Our second step is to get a solution to (2). There is a very nice solution for this equation, that can be obtained using Brown-Crothers methods of [8]. In this reference can be consulted the implied boundary value problems. For this reason we call this the “Brown-Crothers representation” of the particular solution to (2). Let´s deduce this solution. We can see that:

$$\breve{G}(\mathbf{x},t;\ \mathbf{x}^{'},t^{'}) = \frac{\delta\left(t-t^{'}-\frac{R}{v}\right)}{R} - \frac{\delta\left(t-t^{'}-\frac{R}{c}\right)}{R} = \int_{\tau(c)}^{\tau(v)}\frac{1}{R}d\delta\big(\tau(z)\big)$$

$$= \int_{\tau(c)}^{\tau(v)}\frac{1}{R}\frac{d}{d\tau(z)}\delta\big(\tau(z)\big)d\tau(z) = \int_{\tau(c)}^{\tau(v)}\frac{d\tau(z)}{R}H\big(\tau(z)\big)\ .$$

Here $H(\tau(z))$ is Heaviside´s function. If we suppose all variables as fixed parameters with z variable we can write: $d\tau(z) = \frac{R}{z^2}dz$ therefore we obtain:

$$\breve{G}(\mathbf{x},t;\ \mathbf{x}^{'},t^{'}) = \int_{c}^{v}\frac{dz}{z^2}H\big(\tau(z)\big)\ , \qquad (3)$$

Which by construction is an explicit particular solution to (2). Now we are going to prove that:

Proposition 4: Every gauge function defined by equation **III**. (1) is a gauge function of the Jackson´s class.

Note: Here we use the name “Jackson´s class” to refer to the class of gauge functions that can be obtained using Jackson´s method sketched in [1].

Proof: Jackson´s equation A-7.3 of [1] becomes when we use (1):

$$\frac{1}{c}\frac{\partial f}{\partial t} = \int\breve{G}(\mathbf{x},t;\ \ \mathbf{x}^{'},t^{'})\varrho(\mathbf{x}^{'},t^{'})dV^{'}dt\ . \qquad (4)$$

We shall call this “Jackson´s condition”, and is the condition that defines the Jackson´s class. Hence, any gauge function is in the Jackson´s class if satisfies (4). Then if we apply the operator ${}_{v}$ to both sides of (4), we can write:

$${}_{v}\frac{1}{c}\frac{\partial f}{\partial t} = \int{}_{v}\breve{G}(\mathbf{x},t;\mathbf{x}^{'},t^{'})\varrho(\mathbf{x}^{'},t^{'})dV^{'}dt$$

$$= -4\pi\frac{c^2-v^2}{(cv)^2}\int\delta(\mathbf{x}-\mathbf{x}^{'})\frac{\partial^2}{\partial t^2}\delta(t-t^{'})\ \varrho(\mathbf{x}^{'},t^{'})dV^{'}dt\ .$$

Here we used equation (2). Now, from Dirac´s delta properties we get:

$${}_{v}\frac{1}{c}\frac{\partial f}{\partial t} = -4\pi\frac{c^2-v^2}{(cv)^2}\frac{\partial^2}{\partial t^2}\varrho(\mathbf{x},t)$$

So:

$$\frac{1}{c}\frac{\partial}{\partial t}\left({}_{v}f + 4\pi\frac{c^2-v^2}{cv^2}\frac{\partial}{\partial t}\varrho(\mathbf{x},t)\right) = 0$$

Hence if we get a solution to our equation (1) of **III** we also solve Jackson´s condition, hence the gauge functions defined by **III.(**1) are in the Jackson´s class. **QED**.

Jackson´s method outlined in [1] is really naive because he started directly from the transformation equation for the scalar potential, and with a lot of faith supposed that all other conditions involved are not really necessary for the determination of the gauge transformation. The method we propose in **III** takes into account all the differential conditions involved in the determination of the gauge function f, so is more "rigorous", and the differential equation for f is deduced under the conditions of certain boundary value problems that restricts f to S^k. In Jackson´s procedure no such differential equation is deduced for the gauge function, so, his class of gauge functions is wider than the one that is determined from equation **III**.(1). However, this equation is a necessary condition, so, if not satisfied no gauge transformation is available. This leads us to suppose that not all functions in the Jackson´s class really define a gauge transformation. Or we are wrong at some point in the proof of proposition (1). However, even if equation **III**.(1) is not necessary its solutions define a gauge transformation. Which is what we requiere. Now Jackson´s method and our method are not really equivalent. It is a miracle that both methods are consistent in the case of the scalar potentials because their consistency in the case of the vector potential requieres strong conditions that are not natural. Let´s show this. We write:

$$\nabla f = \int \breve{G}(\mathbf{x}, t;\ \mathbf{x}', t')\mathbf{J}(\mathbf{x}', t')dV' dt\,. \tag{5}$$

Just like demand Jackson´s method. Now we obtain:

$$_v\,\boldsymbol{\nabla} f = \int {}_v\,\breve{G}(\mathbf{x}, t;\ \mathbf{x}', t')\mathbf{J}(\mathbf{x}', t')\,dV' dt$$

$$= -4\pi\frac{c^2 - v^2}{(cv)^2}\int \delta(\mathbf{x} - \mathbf{x}')\frac{\partial^2}{\partial t^2}\delta(t - t')\,\mathbf{J}(\mathbf{x}', t')dV' dt$$

So:

$$_v\,\boldsymbol{\nabla} f = -4\pi\frac{c^2 - v^2}{(cv)^2}\frac{\partial^2}{\partial t^2}\mathbf{J}(\mathbf{x}, t)\,. \tag{6}$$

This is not an identity, hence must be a new condition on the gauge function. Therefore if we suppose that equation **III. (**1) is valid we adjoin it to (6) as another condition on the gauge function. From these two equations we obtain:

$$\frac{\partial}{\partial t}\left(-\boldsymbol{\nabla}\varrho(\mathbf{x}, t) + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{J}(\mathbf{x}, t)\right) = 0\,. \tag{7}$$

Again, this is not an identity, but a condition between the matter fields. With the help of the continuity equation we can deduce a differential equation that must be satisfied by one of the matter fields. So in order that Jackson´s method, when applied to the gauge transformation of the vector potential, and our method gets consistency the very strong condition (7) must be satisfied, but (7) is not a general condition valid for any electromagnetic field. So, the results of the methods are not coextensive in general.

6. BOUNDARY CONDITIONS FOR MAXWELL'S EQUATIONS ON SIMULTANEITY SPACES

In this section we are going to prove in outline that the Cauchy problem, for Maxwell equations, changes when we change the geometry of space-time. It is well known (see [13]), that if a relation between space and time is established then there are surfaces of discontinuity for the solutions of the Cauchy problem for hyperbolic equations. The underlying physical idea grounding the postulation of this relation, is that there are waves propagating with finite speed on space and time. So, if we suppose such a relation in advance we shall obtain from Maxwell equations certain propagating phenomena, but more importat, we shall obtain a particular geometry: the Minkowski geometry. In this section we are going to suppose that there is no relation between space and time in order to define the Cauchy problem for a "classical space-time", like the ones defined in [17] (p. 249).

To define the Cauchy problem for Maxwell´s equations is not an easy task. At first sight it seems that the problem is well defined because the equations for the fields are hyperbolic. Therefore all we require to get a well defined Cauchy problem is to define the values of the fields and its first time derivates inside the characteristic surfaces. Fock in [13] shows that this point of view is quite correct when, and only when, we suppose that space and time are related by: $t - t_0 = F(\mathbf{x} - \mathbf{x_0})$. Then, the first time derivatives of the fields are undefined over the surfaces defined by:$1 - |\nabla F|^2 = 0$, which are the characteristic surfaces of hyperbolic equations or wave fronts. So the Cauchy problem is meaningless along the wave front. However this is a consequence of one postúlate that is not necessary if we are not willing to consider the special theory of relativity, with its mixing of space and time. We can also start with a simpler condition like $t = t_0$ so the Cauchy problem is defined all along a space of simultaneity and the fields propagate with infinite speed. So our propagation problem depends on the space-time geometry and a series of assumptions not logically derived from Maxwell´s equations themselves, but only compatible with them. All this information is contained in the geometry of space-time, but also on the Green´s functions, which is the formalism we have been using. Even more, if we define our Cauchy problem with the condition $t = t_0$ only, the Cauchy data are spread along all the simultaneity spaces of our space-time, which is in contrast with the case: $t - t_0 = F(\mathbf{x} - \mathbf{x_0})$ where the Cauchy data are defined only on the interior of the wave front. This case can be studied in [13], so let us determine the Cauchy data for the simultaneity spaces to prove our assertion that they are non-local in space. So we take:

$$\mathbf{E}(\mathbf{x}, t = t_0) = \mathbf{E}_0(\mathbf{x})$$

$$\frac{\partial}{\partial t}\mathbf{E}(\mathbf{x}, t = t_0) = \mathbf{E}_1(\mathbf{x})$$

$$\mathbf{B}(\mathbf{x}, t = t_0) = \mathbf{B}_0(\mathbf{x})$$

$$\frac{\partial}{\partial t}\mathbf{B}(\mathbf{x}, t = t_0) = \mathbf{B}_1(\mathbf{x})$$

As our Cauchy data. This problem is not pointwise defined, but all along the space D where a static electromagnetic field is present. Clearly in the case $t - t_0 = F(\mathbf{x} - \mathbf{x_0})$ the field is defined only inside the wave front. Now, Maxwell´s equations for the Cauchy data are:

$$\left.\begin{aligned} \nabla\cdot\mathbf{E}_0 &= 4\pi\varrho_0 \\ \nabla\times\mathbf{E}_0 &= -\frac{1}{c}\mathbf{B}_1 \\ \nabla\cdot\mathbf{B}_0 &= 0 \\ \nabla\times\mathbf{B}_0 &= \frac{1}{c}\mathbf{E}_1 + \frac{4\pi}{c}\mathbf{J}_0 \end{aligned}\right\}, \tag{1}$$

Where: $\mathbf{J}_0 = \mathbf{J}(\mathbf{x}, t = t_0)$, $\varrho_0 = \varrho(\mathbf{x}, t = t_0)$. Now, if we want to know the discontinuities of the Cauchy data for time derivatives $\mathbf{E}_1, \mathbf{B}_1$ we write the equations (1) in the form:

$$\left.\begin{aligned} \nabla\cdot\mathbf{E}_1 &= -4\pi\nabla\cdot\mathbf{J}_0 \\ -\frac{1}{c}\nabla\times\mathbf{B}_1 &= 4\pi\nabla\varrho_0 - \Delta\mathbf{E}_0 \\ \nabla\cdot\mathbf{B}_1 &= 0 \\ \frac{1}{c}\nabla\times\mathbf{E}_1 &= -\Delta\mathbf{B}_0 - \frac{4\pi}{c}\nabla\times\mathbf{J}_0 \end{aligned}\right\}. \tag{2}$$

Now we can apply Helmholtz theorem (nice explanation on [14], and its use on the case of the veolicty gauge in [15]) on the vector fields $\mathbf{E}_1$, $\mathbf{B}_1$ to obtain:

$$\begin{aligned} \frac{\partial}{\partial t}\mathbf{E}(\mathbf{x}, t = t_0) = -c\nabla\times\left[\int G_l(\mathbf{x}, t; \mathbf{x}', t')\left(\Delta'\mathbf{B}_0(\mathbf{x}') + \frac{4\pi}{c}\nabla'\times\mathbf{J}_0(\mathbf{x}')\right)dV'dt'\right] \\ -4\pi\nabla\cdot\left(\int G_l(\mathbf{x}, t; \mathbf{x}', t')\nabla'\cdot\mathbf{J}_0(\mathbf{x}')dV'dt'\right), \end{aligned} \tag{3}$$

$$\frac{\partial}{\partial t}\mathbf{B}(\mathbf{x}, t = t_0) = -c\nabla\times\left(\int G_l(\mathbf{x}, t; \mathbf{x}', t')(\Delta'\mathbf{E}_0(\mathbf{x}') - 4\pi\nabla'\varrho_0(\mathbf{x}'))dV'dt'\right). \tag{4}$$

There are some technical points here. The Cauchy data must be twice continously differentiable and these derivatives must be Lebesgue integrable in order to get sense from the exchange of limiting processes and integral signs. We suppose in the following that these conditions are met. From (3)-(4) is clear that the only points in infinite space where we can find a discontinuty are located at the origin, i.e., at the poles of the Green´s function, but out these poles the derivatives are continous. Of course, sometimes it is posible to use a Helmholtz theorem for finite spaces, but in this case the discontinuities will be located, probably, on the surfaces limiting the space (see [17]). We can see also that the Cauchy data are globally defined all along the space of simultaneity, not just along a wave front. Now we must note that the case $t = t_0$ is imposible in the STR because implies simultaneity of events, so, the geometry of space-time is quite different in this case from the geometry of Minkowski space-time. Hence Maxwell equations are clearly elliptic equations for a classical space-time where simultaneity spaces are allowed. So the Coulomb gauge, where the equation for the scalar potential is elliptic, is very different from any gauge involving finite speeds.

7. Proof of Local Existence of Gauge Transformations

In this section we are going to prove that gauge transformations from the velocity gauge to the Lorenz gauge are no globally defined, but they are always local in nature. From **VI** we learned that the geometry of space-time determines the Cauchy problem. In the case of a space-time accepting simultaneity spaces the Cauchy data are well defined all along the space of simultaneity, so the fields are essentially described by elliptic equations, while in the case of the existence of wave fronts the equations must be hyperbolic, as is dictated by accepted wisdom. However in the Lorenz and velocity gauges the space-times are minkowskian, so the previous situation is irrelevant. We are shall prove that the gauge functions are not globally defined for this case and explain the meaning of this statement.

So we have:

Proposition 5: The gauge function transforming the Lorenz gauge into the velocity gauge is not globally defined, i.e., it is not defined all along the space-time.

Proof: We just need the equations.

$$\frac{1}{c}\frac{\partial f}{\partial t} = \int \breve{G}(\mathbf{x},t;\ \mathbf{x}',t')\varrho(\mathbf{x}',t')dV'\,dt\,, \tag{1}$$

$$\nabla f = \int \breve{G}(\mathbf{x},t;\ \mathbf{x}',t')\mathbf{J}(\mathbf{x}',t')dV'\,dt\,, \tag{2}$$

$$\breve{G}(\mathbf{x},t;\ \mathbf{x}',t') = \int_c^v \frac{dz}{z^2} H\big(\tau(z)\big)\,. \tag{3}$$

So, if $\tau(z) \le 0$ then $H\big(\tau(z)\big) = 0$, hence, because all the first derivatives of f are zero, no gauge transformation exists for the family of sets:

$$N_z(\mathbf{x}',t') = \left\{\langle x,y,z,t\rangle \in R^4 \,\middle|\, t - t' - \frac{R}{z} \le 0\right\}, c \le z \le v\,. \tag{4}$$

QED

For instance consider N_c. So we have: $c \le \frac{R}{t-t'}$ and all the points in N_c are outside the light cone, so they are not local. We call these points “space-like” points. This is also the case for N_v, so the points where there is no gauge transformation are those outside the set N_c, i.e., all the space-like points. So there is no gauge transformation between the velocity and Lorenz gauges for all space-like points. The gauge transformation exists only inside the light cone. Hence only where the velocity c is equal to v there is a gauge transformation relating the solutions. So we cannot expect to violate STR with the help of the velocity gauge.

8. Conclusions

Within the span of this paper we have obtained the following results:

1. A new method for obtaining gauge transformation functions that is not equivalent to Jackson´s.

2. A proof that gauge transformations functions are local transformations, i.e., there are some space-time sets where these transformations doesn´t exists. This result is valid for the Lorenz and velocity gauge, for the case of the Coulomb gauge and any gauge involving wave fronts we have and indirect result.

3. The Cauchy problem for Maxwell equations depends on the geometry of space-time. So it is quite impossible an equivalence of solutions with the help of a gauge transformation because in the case of hyperbolic equations there is always a wave front, while in the case of elliptic equations no such wave front exists. Hence in one case the solutions allow singularities not allowed in the other case.

The consequences for electromagnetic theory, at least for its ideology, is that gauge transformations cannot be used to obtain global equivalence, but only a local equivalence between the solutions of any gauge. Hence, the Lorenz gauge is locally equivalent to the V-gauge. This was outlined by Brown and Crothers in [8] but their proof was flawed.

Now, about the faster than light particles we think that they are possible in classical space-times because from the start an instantaneous static field is defined with the help of the Cauchy data. Form equations **VI.** (3)-(4) is quite clear that if this static electromagnetic field is zero then the presence of charges is enough to spread the time derivatives all along the space. If the matter fields are zero the time derivatives are always zero. So, no propagation exists for simultaneity spaces.

APPENDIX

The scalar potentials in each gauge are given, in the infinite domain, by:

$$\varphi(\mathbf{x},t) = \int G_L(\mathbf{x},t;\mathbf{x}',t')\varrho(\mathbf{x}',t')dV'dt' \qquad (A1)$$

$$\varphi_v(\mathbf{x},t) = \int G_\alpha(\mathbf{x},t;\mathbf{x}',t')\varrho(\mathbf{x}',t')dV'dt'. \qquad (A2)$$

and the vector potentials by:

$$\mathbf{A}(\mathbf{x},t) = \int G_L(\mathbf{x},t;\mathbf{x}',t')\mathbf{J}(\mathbf{x}',t')dV'dt' \ ,$$

$$\mathbf{A}_v(\mathbf{x},t) = \int G_\alpha(\mathbf{x},t;\mathbf{x}',t')\mathbf{J}(\mathbf{x}',t')dV'dt' \ .$$

With Green´s funciones given by:

$$G_L(\mathbf{x},t;\mathbf{x}',t') = \frac{\delta\left[t-\left(\frac{|\mathbf{x}-\mathbf{x}'|}{c}-t'\right)\right]}{|\mathbf{x}-\mathbf{x}'|} \ , \qquad (A3)$$

$$G_\alpha(\mathbf{x},t;\mathbf{x}',t') = \frac{\delta\left[t-\left(\frac{|\mathbf{x}-\mathbf{x}'|}{\alpha c}\right)-t'\right]}{|\mathbf{x}-\mathbf{x}'|}, \qquad (A4)$$

$$G_L(\mathbf{x},t;\mathbf{x}',t') = -4\pi\delta(\mathbf{x}-\mathbf{x}')\delta(t-t'), \qquad (A5)$$

$$_vG_\alpha(\mathbf{x},t;\mathbf{x}',t') = -4\pi\delta(\mathbf{x}-\mathbf{x}')\delta(t-t'), \qquad (A6)$$

$$\frac{\partial}{\partial t}G(\mathbf{x},t;\mathbf{x}',t') = -\frac{\partial}{\partial t'}G(\mathbf{x},t;\mathbf{x}',t'), \qquad (A7)$$

$$G_l(\mathbf{x},t;\mathbf{x}',t') = \frac{1}{4\pi}\frac{\delta(t-t')}{|\mathbf{x}-\mathbf{x}'|} \ . \qquad (A8)$$

The formula (A7) is valid for whatever Green´s function, so we skip the sub-index. Much more details can be found in chapter IV of [12].

For the sake of simplicity we introduce the following inner product in our functional space:

$$\langle \varphi(\mathbf{x}', t')|\mu(\mathbf{x}', t')\rangle = \int \varphi(\mathbf{x}', t')\mu(\mathbf{x}', t')dV'dt' . \quad (A9)$$

All the integrals are taken over all space-time except if otherwise stated. Our space-time is euclidean, being all positions defined with the help of an inertial cartesian coordinate system.

REFERENCES

[1] Jackson J D 2002 From Lorenz to Coulomb and other explicit gauge transformations *Am. J. Phys.* **70** (9) 917

[2] Jackson J D and Okun L B 2001 Historical Roots of Gauge Invariance *Rev. Mod. Phys.* **73** 663-680

[3] Recami E 1986 Classical Tachyons and Possible Applications *Rivista Del Nuovo Cimento* **9** 1-178

[4] Bilaniuk O, Desphande V and Sudarshan E 1962"Meta"- relativity *Am. J. Phys.* **30** 718

[5] Feinberg G 1967 Possibility of faster than light particles *Phys. Rev.* **159** (5) **1089**

[6] Robinett L 1978 Do Tachyons travel more slowly than light *Phys. Rev. D* **18** 3610

[7] Kiskis J 1978 Disconnected gauge groups and the global violation of charge conservation Phys. *Rev. D* **17** (12) 3196

[8] Brown G and Crothers D 1989 Generalised gauge invariance of electromagnetism *J. Phys a: Math and Gen* **22** 2939

[9] Yang K H 2005 the Physics of Gauge Transformations *Am. J. Phys.* **73** (8) 742

[10] Yang K H 1976 Gauge transformations and quantum mechanics I *Ann. Phys* **101**(1) 97

[11] Sobolev S L 1989 *Partial Differential Equations* (Dover, NJ)

[12] Barton G 1989 *Elements of Green´s Functions and Propagation* (Oxford, UK)

[13] Fock V 1964 *the Theory of Space-Time and Gravitation* (Pergamon Press Ltd.)

[14] Heras R 2016 the Helmholtz theorem and retarded fields Eur *J Phys* **37** (6) 1-11 doi:10.1088/0143-0807/37/6/065204

[15] Chubykalo A, Espinoza A, Alvarado Flores R and Gutiérrez Rodríguez A 2006 Helmholtz theorem and the V-gauge in the Problem of Superluminal Interactions *Foundations of Physics Letters* **19** (1) 37-49

[16] Malament D. 2012 *Topics in the Foundations of General Relativity and Newtonina Gravitation Theory* (Chicago)

[17] Kustepeli A 2016 on the Helmholtz Theorem and its Generalization for Multilayers *Electromagnetics* **37** (3) 135

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