Reply to "Comment on 'Action at a distance as a full-value solution of Maxwell equations: The basis and application of the separated-potentials method'"

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The preceding Comment criticized the necessity of introducing an *electrodynamics dualism concept* based on a novel complete solution of Maxwell's equations proposed in our previous paper [Phys. Rev. E **53**, 5373 (1996)]. All arguments made by the authors of the Comment to demonstrate the adequacy of the usual Liénard-Wiechert retarded solutions for a consistent description of electromagnetic phenomena are shown to be invalid beyond the context of boundary conditions for the inhomogeneous D'Alembert equation. From a reinterpretation of Villecco's work [Phys. Rev. E **48**, 4008 (1993)] we concluded that it cannot be applied directly to refute our results and, in contrast to the opinion of the authors of the Comment, it can be used instead to support our claims of mathematical deficiency and inadequacy of Liénard-Wiechert retarded solutions. [S1063-651X(98)12402-9]

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In the preceding Comment [1], Ivezić and Škovrlj argued why in their opinion the *separated-potential method* and *electrodynamics dualism concept* (introduced in our paper [2]) are not justified and, as a consequence, that all proposed modifications to the standard interpretation of classical electrodynamics as a whole are meaningless. It seems that a good critique cannot be made without considering the central part of the method or approach discussed. Nevertheless, the authors of the Comment seemed to find this requirement irrelevant. Their reasoning is centered mainly around the initial part of our analysis concerning the illustrative example, the omission of which would have no consequence on the understanding and rigor of the subsequent material. Even in discussing the issue they have chosen, the authors of the Comment in our opinion did not demonstrate consistently why Liénard-Wiechert (LW) potentials are adequate for a correct description of the properties of total electromagnetic field along the direction of an arbitrarily moving charge. The *paradoxical situation* that we illustrated in the introductionary part remains an open question in the framework of conventional electrodynamics. In fact, the Poynting vector represents a real flux of electromagnetic energy, and from the accepted point of view is the unique mechanism associated with the change of field components for every space point in any direction. However, neither *longitudinal* nor *transverse* components of *LW potentials* can contribute to the Poynting vector along the direction of motion of the charge, and therefore there is no flux of electromagnetic energy associated with the change of field components in this direction. Can this be considered a consistent description related to *LW potentials*?

On the other hand, the traditional theory looks at longitudinal field components as "*unphysical*" solutions. The quantization of the Maxwellian electromagnetic field in QED leads to the notion of the massless photon as a quantum oscillator of exclusively transverse nature. Inclusion of the longitudinal component into photon structure would imply a nonzero photon mass and would contradict all traditional concepts of quantum electrodynamics. The *condition of transversality* is imposed in the conventional theory by the *Coulomb* or *transverse gauge*. This corresponds exactly to the fields-only approach developed by Donnelly and Ziolkowski [3]. Contrary to the interpretation of this result given by the authors of the Comment, the instantaneous longitudinal electric field is canceled exactly by the term contained in the transverse component due to the additional condition imposed *a priori* and not due to the intrinsic property of this component. If we turn to the work of Donnelly and Ziolkowski, we immediately find the following [3]: "*. . . any changes in* $\varrho(\mathbf{r},t)$ *are manifested instantaneously throughout space in* $\mathbf{E}(\mathbf{r},t)$. *Nevertheless, if we impose the condition that the effects of both source terms . . . be propagated in a retarded sense in space-time, then . . . .*" This imposed condition can be interpreted to be equivalent to enforcing the Coulomb or the transverse gauge in the conventional theory and explains why the fields-only approach dispenses explicitly with the need for gauge conditions. Thus, in both approaches instantaneous longitudinal components are eliminated, which allows the total electric field to be propagated in a retarded fashion.

The part of the arguments presented by the authors of the Comment that needs to be addressed the most is based on a rather vague interpretation of Villecco's work [4]. They claim, for example, that there may be only one complete solution represented in different but equivalent forms, in LW retarded-time representation or in an instantaneous action at a distance format. Following their straightforward conclusion, the 19th century opposition between *Maxwellian* and *Newtonian* schools of thought might be perceived as a disappointing delusion. However, the underlying significance of Villecco's work is quite different. If the mathematical formalism developed in [4] is correct, it means only that every source function $\varrho'(t')$ at the retarded time t' used usually

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for LW solutions can be substituted by some effective source function $\varrho'(t)$ at present time t. Additionally, it must be noted that this effective source function $\varrho'(t)$, generally speaking, has nothing to do with the real distribution of charges and currents at present time t. Only in the static limit and for uniformly moving charge does the effective function $\varrho'(t)$ coincide with real source function $\varrho(t)$ (which is not surprising since in this situation one is always dealing exclusively with instantaneous action at a distance, i.e., there is no place for retardation effects) and, as a result, these are the only cases when Villecco's instantaneous action at a distance coincides with the real instantaneous action at a distance tied to the present localization of charges and currents. Thus, the claim that there is always one complete solution (which can be presented in different but equivalent forms) is invalid when applied to our results [2] where we related longitudinal components only with the real action at a distance.

Nevertheless, there is an interesting aspect of Villecco's results that confirms our analysis. In Villecco's approach there appears a so-called *causality parameter* α, which determines respective weights of retarded and advanced potentials in the resulting composite solution. For example, $\alpha = -1$ corresponds entirely to the LW retarded-time components whereas $\alpha = 1$ only to the advanced-time components. The additional advantage of Villecco's approach over the conventional one is that it reveals a mathematical deficiency of commonly accepted LW solutions that he resumed in the following manner [4]: ''... If $\alpha \neq 0$, the transition between two different states of uniform velocity via an intermediate state of acceleration results in a type of discontinuity in functional form... Though no known law is violated in this processes, there is a sense of intrinsic continuity which is nevertheless violated.'' The same conclusion was expressed in a different way in our work [2]: ''...the conventional theory is unable to describe correctly the transition from a uniform movement of a charge into an arbitrary one and then again into uniform over a limited interval of time. In this case, the first and the latter solutions can be given exactly by the Lorentz transformation. Furthermore the question arises: what mechanism changes these potentials at the distance unreachable for retarded Liénard-Wiechert fields? The lack of continuity between the corresponding solutions is obvious ....'' This mathematical deficiency of LW potentials has been related in our work with the incompleteness of existent solutions (below we will look at this problem in more detail).

The problem of the intrinsic continuity of electromagnetic phenomena is the starting point of our central reasoning, which apparently was ignored by our critics. In Sec. III, entitled ''*Reasons and Foundations of the Method of Separated Potentials*,'' we begin the discussion by wondering if continuous transitions between steady-state and arbitrary time-varying problems are ensured within the framework of the conventional electrodynamics. This formulation turns out to be closely connected with the mathematical analysis of initial and boundary value conditions required by force of the uniqueness theorem for selecting unique and adequate solution (from the infinite number of solutions admitted for every differential equation).

In this respect, the straightforward statement made by the authors of the Comment that LW potentials as a full-value solution of D'Alembert equations (an equivalent from of Maxwell's equations) must be *a priori* adequate for a complete description of electromagnetic phenomena has no solid ground beyond the context of boundary conditions. In other words, in spite of the fact that LW potentials are a class of full-value solutions of D'Alembert's equations (within a more general class of full-value solutions related to a different formulation of the initial Cauchy problem) and satisfy them in all directions (as well as in the direction of a moving charge that we also recognized in the erratum [2]), their adequacy for the consistent description of the total electromagnetic field must be established independently in a rigorous manner taking into account additional arguments.

This point appears not to have been realized by the authors of the Comment and explains the lack of discussion about the *method of separated potentials* introduced for the purpose of making the whole electromagnetic description self-consistent. In order to be brief, we will not repeat the reasoning exposed in Sec. III [2] but only emphasize that a modification in the formulation of the initial Cauchy problem by the inclusion of our condition (iii) concerning the uniform convergence of the general solution to zero at infinity eliminates the above-mentioned mathematical deficiency of the existent LW solutions related to the lack of continuity with respect to the transition between steady and arbitrary time-varying processes in the conventional theory. The novel boundary condition apparently differs from the usual condition for the initial Cauchy problem established to obtain LW potentials. This results in a different form of the proposed solution: its structure consists of two orthogonal (nonreducible) functions with implicit and explicit time dependence that is obviously opposed to the structure of LW potentials understood exclusively as explicit time-dependent solution. The absence of the instantaneous longitudinal component (equivalent to instantaneous action at a distance) in a set of commonly accepted solutions of D'Alembert equations is, in our opinion, an indication of their incompleteness. On the other hand, contrary to what would seem to be the case at first glance, these longitudinal components are compatible with the principle of relativity and satisfy all requirements on relativistic invariance. Thus, the introduction of *electrodynamic dualism concept* (simultaneous coexistence of instantaneous longitudinal long-range and Faraday-Maxwell short-range interactions) is based on solid mathematical grounds and it has not been refuted by the authors of the Comment.

Additionally, it might be noted that boundary conditions have never been analyzed in the conventional approach from the point of view of relativity. Though the initial Cauchy problem for electromagnetic phenomena had been formulated independently by Liénard [5] and Wiechert [6] several years before the appearance of Einstein's theory in 1905, it has not been tested for relativistic invariance. The possible difficulty with the existent boundary conditions for D'Alembert equations was realized by Einstein himself a few months before his death in 1955. In the last edition of *Meaning of Relativity* he stated [7]: ''... A field theory is not yet completely determined by the system of field equations ... Should one postulate boundary conditions? ... Without such a postulate, the theory is much too vague. In my opinion the answer to the question is that postulation of boundary conditions is indispensable.'' Thus, Einstein himself indicated a new direction for forthcoming investigations but could not pursue this important proposal because of his death. This open problem still remains unresolved today. Although this discussion of the fundamentals of field theory is obviously out of the scope of this Reply, it is appropriate to note that our boundary condition is a real candidate to fulfill the requirements of relativistic invariance. For example, potentials of one uniformly moving charge are functions with a regular behavior at infinity. This agrees with the condition (iii), which requires Lorentz invariance. A rigorous analysis is planned to be effected elsewhere.

In order to avoid some possible misunderstanding of the question about a mechanism that interconnects two nonreducible components of our complete solution (this is obviously important in the study of radiation processes), we draw attention to the equivalent representation of an uncoupled pair of inhomogeneous D'Alembert equations in the form of two pairs of second order differential equations for each component of the general solution (see formulas (21)–(24) in [2]):

$$\Delta\varphi_0 = -4\pi\varrho(\mathbf{r},t), \quad \Delta\mathbf{A}_0 = -\frac{4\pi}{c}\mathbf{j}(\mathbf{r},t) \tag{1}$$

and

$$\Delta\varphi^* - \frac{1}{c^2}\frac{\partial^2\varphi^*}{\partial t^2} = 0, \quad \Delta\mathbf{A}^* - \frac{1}{c^2}\frac{\partial^2\mathbf{A}^*}{\partial t^2} = \mathbf{0} \tag{2}$$

with the following initial and boundary conditions (given, for instance, in the case of the electric potential). The first equation in (1), apart from the condition (iii), is also supplemented by

$$\varphi_0(\mathbf{r})|_S = G, \tag{3}$$

whereas the first equation in (2) must be added with

$$\varphi^*(\mathbf{r},t)|_{(t=0)} = G_1 - \varphi_0(\mathbf{R}(t))|_{(t=0)}, \tag{4}$$

$$\frac{d}{dt}[\varphi^*(\mathbf{r},t)]\bigg|_{(t=0)} = G_2 - \frac{d}{dt}[\varphi_0(\mathbf{R}(t))]\bigg|_{(t=0)}, \tag{5}$$

where functions $G_1(\mathbf{r})$ and $G_2(\mathbf{r})$ have been defined for the common Cauchy problem (see [2]).

Jointly, Eqs. (1) and (2), on one hand, and conditions (3)–(5), on the other, are equivalent to the inhomogeneous D'Alembert equations with fully established boundary conditions. Therefore, since a complete solution of inhomogeneous D'Alembert equations is formed by a general solution of homogeneous equation plus some particular solution of the inhomogeneous one, we assume that the same must be satisfied by the equivalent form (1),(2). In this case a complete solution is constructed, on one hand, from two independent general solutions satisfying homogeneous Poisson's and homogeneous wave equations, respectively, and, on the other hand, from one particular solution [as a linear combination of two nonreducible components $(\varphi_0,\mathbf{A}_0)$ and $(\varphi^*,\mathbf{A}^*)$], satisfying inhomogeneous D'Alembert equations. Mutual relation between both components of electromagnetic field is dictated by Eqs. (4) and (5) and is enclosed in the particular solution of an inhomogeneous D'Alembert equation. More comprehensive study of the matter must be made elsewhere. It would be necessary in order to describe correctly a detailed energy balance between two subsystems corresponding to respective nonreducible energy-carrying components $(\varphi_0,\mathbf{A}_0)$ and $(\varphi^*,\mathbf{A}^*)$ of the total electromagnetic field.

Turning to some properties of the proposed complete solution of Maxwell's equations, the following remarks can be made. First, condition (iii) cannot be removed from the formulation of the initial Cauchy problem that results in the fundamental (nonremovable) nature of the implicit time-dependent component responsible for interparticle long-range (Coulomb-type) interaction. Otherwise, the continuous transition from the initial Cauchy problem into an external boundary-value problem for Poisson's equation is not ensured and, as a result, mutual continuity between the corresponding solutions cannot be expected by force of the uniqueness theorem. Thus, intrinsic nonlocality properties of classical electromagnetism appear in our approach in the most natural way. In this respect, it might be noted that during the last few decades modern physics has been faced with fundamental difficulties in unifying classical physics elaborated upon within the framework of the locality concept of relativistic theory on one hand and quantum physics on the other. The latter is characterized essentially by the emergence of nonlocality, e.g., violation of Bell's inequalities, Aharonov-Bohm effect, etc. *This significant incommensurability* between both theories must lead, according to Bohm, to the discovery of an entirely new order to physics at a fundamental level [8]. There is currently no rigorous mutual correspondence between these two fundamental areas. On the contrary, the proposed *dualism concept* might put any classical system at the same fundamental level in regard to inseparability and nonlocality, as these two properties are accepted in quantum mechanics systems. In this case instantaneous action at a distance as represented by longitudinal field components can be interpreted as a classical equivalent of nonlocal quantum interactions.

In conclusion, we would like to emphasize once more the initial proposal of the discussed work. Internal difficulties arose during the development of classical electrodynamics. The last serious attempts to make the electromagnetic theory satisfactory had been effected in the middle of this century. Nevertheless, since then the situation has not changed. To be more specific, we turn to R. Feynman who writes [9]: ''...this tremendous edifice (classical electrodynamics), which is such a beautiful success in explaining so many phenomena, ultimately falls on its face. When you follow any of our physics too far, you find that it always gets into some kind of trouble. ...the failure of the classical electromagnetic theory. ...Classical mechanics is a mathematically consistent theory; it just doesn't agree with experience. It is interesting, though, that the classical theory of electromagnetism is an unsatisfactory theory all by itself. There are difficulties associated with the ideas of Maxwell's theory which are not solved by and not directly associated with quantum mechanics . . . .'' In this respect, as has been discussed in the second part of our paper [2], some unexpected properties of our proposed complete solution of Maxwell's equations turn out to be capable of removing all principal inconsistencies from classical electrodynamics and thereby ought to be studied more carefully with reasonable caution but without prejudice.

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