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Dipole moments of the tau-lepton and Z–Z' mixing angle induced in a 331 model

A Gutiérrez-Rodríguez[1], **M A Hernández-Ruíz**[2] **and C P Castañeda-Almanza**[3]

[1] Facultad de Física, Universidad Autónoma de Zacatecas, Apartado Postal C-580, 98060 Zacatecas, Mexico

[2] Unidad Académica de Ciencias Químicas, Universidad Autónoma de Zacatecas, Apartado Postal C-585, 98060 Zacatecas, Mexico

[3] Facultad de Física, Universidad Autónoma de Zacatecas, Apartado Postal C-580, 98060 Zacatecas, Mexico

E-mail: alexgu@fisica.uaz.edu.mx, mahernan@uaz.edu.mx and cosmycastaneda@fisica.uaz.edu.mx

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Abstract

Using as an input the data obtained by the L3 and OPAL Collaborations for the reaction $e^+e^- \to \tau^+\tau^-\gamma$ at the Z-pole, we obtained bounds on the electromagnetic and weak dipole moments of the tau-lepton in the context of a 331 model. Our bounds on the electromagnetic moments are consistent with the bounds obtained by the L3 and OPAL Collaborations for the reaction $e^+e^- \to \tau^+\tau^-\gamma$. We also obtained bounds on the tau weak dipole moments which are consistent with the bounds obtained recently by the DELPHI, ALEPH and BELLE Collaborations from the reaction $e^+e^- \to \tau^+\tau^-$. In addition, we obtained a limit on the Z–Z' mixing angle of the 331 model: $-3.92 \times 10^{-3} \leqslant \phi \leqslant 1.30 \times 10^{-4}$, which is competitive with those reported in the literature. Our work complements other studies on the electromagnetic and weak dipole moments of the tau-lepton and on the Z–Z' mixing angle.

1. Introduction

The production of tau-lepton pairs in high energy e^+e^- collisions has been used to set bounds on its electromagnetic and weak dipole moments [1–7]. In the Standard Model (SM) [8–10], the τ anomalous magnetic moment (MM) $a_\tau = (g_\tau - 2)/2$ is predicted to be $(a_\tau)_{SM} = 0.0011773(3)$ [11, 12] and the respective electric dipole moment (EDM) d_τ is generated by the GIM mechanism only at very high order in the coupling constant [13]. Similarly, the weak MM and EDM are induced in the SM at the loop level giving $a^W_\tau = -(2.10 + 0.61\mathrm{i}) \times 10^{-6}$ [14, 15] and $d^W_\tau \leqslant 8 \times 10^{-34} e$ cm [16, 17]. Since the current bounds on these dipole moments [6, 7, 3, 4] are well above the SM predictions, it has been pointed out that these quantities are excellent candidates to look for physics beyond the SM [14–26]. The couplings of the photon and Z gauge boson to charged leptons may be parameterized in the following form:

$$\Gamma_V^\alpha = eF_1(q^2)\gamma^\alpha + \frac{\mathrm{ie}}{2m_l}F_2(q^2)\sigma^{\alpha\mu}q_\mu + eF_3(q^2)\gamma_5\sigma^{\alpha\mu}q_\mu, \tag{1}$$

where $V = \gamma, Z$, m_l is the lepton mass and $q = p' - p$ is the momentum transfer. The q^2-dependent form-factors $F_i(q^2)$ have familiar interpretations for $q^2 = 0$: $F_1(0) \equiv Q_l$ is the electric charge; $F_2(0) \equiv a_l$; and $F_3 \equiv d_l/Q_l$. The weak dipole moments are defined in a similar way: $F_2^Z(q^2 = m_Z^2) = a_\tau^W$ and $F_3^Z(q^2 = m_Z^2) = d_\tau^W/e$. The measurement of a_τ^W and d_τ^W has been done in the $Z \to \tau^+\tau^-$ decay mode at LEP. The latest bounds obtained for the electromagnetic and weak dipole moments from the DELPHI, ALEPH and BELLE Collaborations at the 95% CL are: $-0.052 < a_\tau < 0.013$, $-0.22 < d_\tau(10^{-16}\ e\ \mathrm{cm}) < 0.45$ and $a_\tau^W < 1.1 \times 10^{-3}$, $d_\tau^W < 0.50 \times 10^{-17} e$ cm [3–5, 27].

The first limits on the MM and EDM of the τ lepton were obtained by Grifols and Méndez using L3 data [22]: $a_\tau \leqslant 0.11$ and $d_\tau \leqslant 6 \times 10^{-16} e$ cm. Escribano and Massó [21] later used electroweak precision measurements to get $d_\tau \leqslant 1.1 \times 10^{-17} e$ cm and $-0.004 \leqslant a_\tau \leqslant 0.006$ at the 2σ confidence level. There is extensive theoretical work done in models beyond the SM that contribute to EDM of charged leptons. In [28], the EDM of charged leptons are studied assuming that they have Gaussian profiles in extra dimensions. In [29] the lepton EDM has been analyzed in the framework of the seesaw model. The electric dipole moments of the leptons in the version III of the 2HDM are considered in [30]. The work [31] was related to the lepton EDM in the framework of the SM with the inclusion of non-commutative geometry. Furthermore, the effects of non-universal extra dimensions on the EDM of fermions in the two Higgs doublet model have been estimated in [32]. In [33–36], limits on the electromagnetic and weak dipole moments of the tau-lepton in the framework of a left–right symmetric model (LRSM) and a class of E_6 inspired models with an additional neutral vector boson Z_θ have been analyzed.

The existence of a heavy neutral (Z') vector boson is a feature of many extensions of the SM. In particular, one (or more) additional $U(1)'$ gauge factor provides one of the simplest extensions of the SM. Additional Z' gauge bosons appear in grand unified theories [37], superstring theories [38], LRSM [39–41] and in other models such as models of composite gauge bosons [42]. In particular, it is possible to study some phenomenological features associates with this extra neutral gauge boson through models with gauge symmetry $SU(3)_C \times SU(3)_L \times U(1)_N$, also called 331 models. These models arise as an interesting alternative to explain the origin of generations. Pisano and Pleitez [43–45] have proposed an model based on the gauge group $SU(3)_C \times SU(3)_L \times U(1)_N$. This model has the interesting feature that each generation of fermions is anomalous, but that with three generations the anomalous canceled. Detailed discussions on 331 models can be found in the literature [43–52].

On the Z peak, where a large number of Z events are collected at e^+e^- colliders, one may hope to constrain or eventually measure the electromagnetic and weak dipole moments of the τ by selecting $\tau^+\tau^-$ events accompanied by a hard photon. The Feynman diagrams which give the most important contribution to the cross section from $e^+e^- \to \tau^+\tau^-\gamma$ are shown in figure 1. The total cross section of $e^+e^- \to \tau^+\tau^-\gamma$ will be evaluated at the Z-pole in the framework of a 331 model. The numerical computation for the anomalous magnetic and the electric dipole moments of the tau is done using the data collected by the L3 and OPAL Collaborations at LEP [1, 2]. We are interested in studying the effects induced by the effective couplings associated to the weak and electromagnetic moments of the tau lepton given in equation (1). For this purpose, we will take the respective anomalous vertices $\tau\tau\gamma$ and $\tau\tau Z$, one at the time, in diagrams 1 and 2 of figure 1.

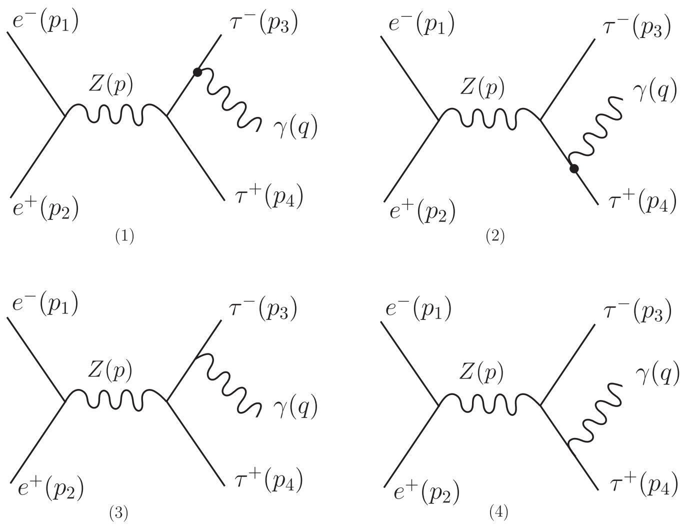

Figure 1. The Feynman diagrams contributing to the process $e^+e^- \to \tau^+\tau^-\gamma$ in a 331 model: anomalous coupling (1, 2) and the SM (3, 4) when the Z vector boson is produced on mass-shell.

In [53], Gau *et al* carry out a comprehensive study on the dipole moments of the tau-lepton. In this paper the authors describe a calculation at tree-level of the process $e^+e^- \to \tau^+\tau^-\gamma$ which accounts for the effects of anomalous magnetic and electric dipole couplings, they consider all the SM and anomalous amplitudes. In the revision and comparison of their results with the simplified result reported in [54], which neglects anomalous contributions from initial-final state interference, from γZ interference, and from γ exchange, they find that the discrepancy is roughly 1% of the total anomalous cross section, showing that one can safely neglect anomalous contributions from initial–final state interference, γZ interference and γ exchange.

Our aim in this paper is to analyze the reaction $e^+e^- \to \tau^+\tau^-\gamma$ in the Z boson resonance. The analysis is carried out in the context of a 331 model [51] and we attribute electromagnetic and weak dipole moments to the tau-lepton. Processes measured in the resonance serve to set limits on the tau electromagnetic and weak dipole moments.

First, using as an input the results obtained by the L3 and OPAL Collaborations [1, 2] for the reaction $e^+e^- \to \tau^+\tau^-\gamma$ at the Z-pole, we obtained bounds on the electromagnetic and weak dipole moments of the tau-lepton in the context of a 331 model. We have found that these limits are consistent with the new bounds obtained by the DELPHI, ALEPH and BELLE Collaborations from the process $e^+e^- \to \tau^+\tau^-$ [3–5]. In addition, we estimate limits on the mixing angle ϕ of the 331 model which is similar to that obtained from the 331 model with right-handed neutrinos [48], as well as that obtained recently from the LEP data on the number of light neutrino species in the 331 model [51].

This paper is organized as follows. In section 2 we present the calculation of the cross section for the process $e^+e^- \to \tau^+\tau^-\gamma$ in a 331 model. In section 3 we present our results for the numerical computations and, finally, we present our conclusions in section 4.

2. The total cross section for the reaction $e^+e^- \to \tau^+\tau^-\gamma$

In this section we calculate the total cross section for the reaction $e^-(p_1)e^+(p_2) \to \tau^-(p_3)\tau^+(p_4)\gamma(q)$ using the neutral current Lagrangian given in equation (9) of [51] for the 331 model for the diagrams 1–4 of figure 1. A characteristic interesting from this model is that is independent of the mass of the additional Z' heavy gauge boson and so we have the mixing angle ϕ between the Z and Z' bosons as the only additional parameter. The respective transition amplitudes are thus given by

$$\begin{aligned}\mathcal{M}_1 = \frac{-g^2}{4\cos^2\theta_W(l^2 - m_\tau^2)} & \left[\bar{u}(p_3)\Gamma_\gamma^\alpha(\not{l} + m_\tau)\gamma^\mu\left(\cos\phi - \frac{\sin\phi}{\sqrt{3 - 4\sin^2\theta_W}}\right)\right. \\ & \left.\times\left(g_V^\tau - g_A^\tau\gamma_5\right)v(p_4)\right] \\ & \times\frac{(g_{\mu\nu} - p_\mu p_\nu/M_Z^2)}{\left[(p_1 + p_2)^2 - M_Z^2 - \mathrm{i}\Gamma_Z^2\right]}\left[\bar{u}(p_2)\gamma^\nu\left(\cos\phi - \frac{\sin\phi}{\sqrt{3 - 4\sin^2\theta_W}}\right)\right. \\ & \left.\times\left(g_V^e - g_A^e\gamma_5\right)v(p_1)\right]\epsilon_\alpha^\lambda(q), \end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{M}_2 = \frac{-g^2}{4\cos^2\theta_W(k^2 - m_\tau^2)} & \left[\bar{u}(p_3)\gamma^\mu\left(\cos\phi - \frac{\sin\phi}{\sqrt{3 - 4\sin^2\theta_W}}\right)\right. \\ & \left.\times\left(g_V^\tau - g_A^\tau\gamma_5\right)(\not{k} + m_\tau)\Gamma_\gamma^\alpha v(p_4)\right] \\ & \times\frac{\left(g_{\mu\nu} - p_\mu p_\nu/M_Z^2\right)}{\left[(p_1 + p_2)^2 - M_Z^2 - \mathrm{i}\Gamma_Z^2\right]}\left[\bar{u}(p_2)\gamma^\nu\left(\cos\phi - \frac{\sin\phi}{\sqrt{3 - 4\sin^2\theta_W}}\right)\right. \\ & \left.\times\left(g_V^e - g_A^e\gamma_5\right)v(p_1)\right]\epsilon_\alpha^\lambda(q), \end{aligned} \tag{3}$$

and for $\mathcal{M}_3$ and $\mathcal{M}_4$

$$\mathcal{M}_3 = \mathcal{M}_1\left(\Gamma_\gamma^\alpha \to \gamma^\alpha\right), \tag{4}$$

$$\mathcal{M}_4 = \mathcal{M}_2\left(\Gamma_\gamma^\alpha \to \gamma^\alpha\right), \tag{5}$$

where Γ_γ^α is the tau-lepton electromagnetic vertex which is defined in equation (1), while $\epsilon_\alpha^\lambda(q)$ is the polarization vector of the photon. l and k stands for the momentum of the virtual tau and antitau respectively.

The MM, EDM and the mixing angle ϕ of the 331 model give a contribution to the differential cross section for the process $e^+e^- \to \tau^+\tau^-\gamma$ of the form:

$$\begin{aligned}\sigma(e^+e^- \to \tau^+\tau^-\gamma) = & \int \frac{\alpha^2}{48\pi}\left[\frac{e^2a_\tau^2}{4m_\tau^2} + d_\tau^2\right]\left(\cos\phi - \frac{\sin\phi}{\sqrt{3 - 4x_W}}\right)^4\left[\frac{1 - 4x_W + 8x_W^2}{x_W^2(1 - x_W)^2}\right] \\ & \times\left[\frac{\left(1 - 4x_W + x_W^2\right)(s - 2\sqrt{s}E_\gamma) + \frac{1}{2}E_\gamma^2\sin^2\theta_\gamma}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]E_\gamma\,\mathrm{d}E_\gamma\,\mathrm{d}\cos\theta_\gamma \\ & + \int\frac{\alpha^2 e}{256\pi}\left[\frac{ea_\tau}{2m_\tau} + d_\tau\right]\left(\cos\phi - \frac{\sin\phi}{\sqrt{3 - 4x_W}}\right)^4 \\ & \times\left[\frac{\left(1 - 4x_W + 8x_W^2\right)^2}{x_W^2(1 - x_W)^2}\right] \\ & \times\left[\frac{9E_\gamma^2\sin^2\theta_\gamma + 12\sqrt{s}E_\gamma - 9s - 4\sqrt{s}E_\gamma\sin^2\theta_\gamma}{\sqrt{s}E_\gamma\sin^2\theta_\gamma\left[\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2\right]}\right]E_\gamma\,\mathrm{d}E_\gamma\,\mathrm{d}\cos\theta_\gamma \\ & + \int\frac{\alpha^3}{16}\left(\cos\phi - \frac{\sin\phi}{\sqrt{3 - 4x_W}}\right)^4\left[\frac{\left(1 - 4x_W + 8x_W^2\right)^2}{x_W^2(1 - x_W)^2}\right]\end{aligned}$$

$$\times \left[\frac{-s^2 + 3s\sqrt{s}E_\gamma - 2sE_\gamma^2 + \frac{1}{2}sE_\gamma^2 \sin^2\theta_\gamma - \frac{3}{4}\sqrt{s}E_\gamma^3 \sin^2\theta_\gamma}{sE_\gamma^2 \sin^2\theta_\gamma \left[\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2\right]} \right]$$
$$\times E_\gamma \mathrm{d}E_\gamma \mathrm{d}\cos\theta_\gamma, \tag{6}$$

where $x_W \equiv \sin^2\theta_W$, $\sin\phi$ and $\cos\phi$ are the sine and cosine of the mixing angle of the 331 model and E_γ, $\cos\theta_\gamma$ are the energy and the opening angle of the emitted photon.

It is useful to consider the smallness of the mixing angle ϕ, as indicated in equation (14), to approximate the cross section in equation (6) by its expansion in ϕ up to the linear term. Such an approximation for deriving the bounds of a_τ and d_τ is more illustrative and easier to manipulate.

For $|\phi| \ll 1$, the total cross section for the process $e^+e^- \to \tau^+\tau^-\gamma$ is given by

$$\sigma(e^+e^- \to \tau^+\tau^-\gamma) = \left(\frac{e^2a_\tau^2}{4m_\tau^2} + d_\tau^2\right)[A + B\phi] + \left(\frac{ea_\tau}{2m_\tau} + d_\tau\right)$$
$$\times[C + D\phi] + E + F\phi + O(\phi^2), \tag{7}$$

where A, B, C, D, E and F are constants which can be evaluated. A explicitly is

$$A = \int \frac{\alpha^2}{48\pi}\left[\frac{1 - 4x_W + 8x_W^2}{x_W^2(1 - x_W)^2}\right]\left[\frac{\left(1 - 4x_W + 8x_W^2\right)(s - 2\sqrt{s}E_\gamma) + \frac{1}{2}E_\gamma^2\sin^2\theta_\gamma}{\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2}\right]$$
$$\times E_\gamma \mathrm{d}E_\gamma \mathrm{d}\cos\theta_\gamma, \tag{8}$$

while B, C, D, E and F are given by

$$B = -\int \frac{\alpha^2}{12\pi}\left[\frac{1 - 4x_W + 8x_W^2}{x_W^2(1 - x_W)^2}\right]\left[\frac{\left(1 - 4x_W + 8x_W^2\right)(s - 2\sqrt{s}E_\gamma) + \frac{1}{2}E_\gamma^2\sin^2\theta_\gamma}{\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2}\right]$$
$$\times\left[\frac{1}{\sqrt{3 - 4x_W}}\right]E_\gamma \mathrm{d}E_\gamma \mathrm{d}\cos\theta_\gamma, \tag{9}$$

$$C = \int \frac{\alpha^2 e}{256\pi}\left[\frac{\left(1 - 4x_W + 8x_W^2\right)^2}{x_W^2(1 - x_W)^2}\right]\left[\frac{-9s + 12\sqrt{s}E_\gamma + 9E_\gamma^2\sin^2\theta_\gamma - 4\sqrt{s}E_\gamma\sin^2\theta_\gamma}{\sqrt{s}E_\gamma\sin^2\theta_\gamma\left[\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2\right]}\right]$$
$$\times E_\gamma \mathrm{d}E_\gamma \mathrm{d}\cos\theta_\gamma, \tag{10}$$

$$D = -\int \frac{\alpha^2 e}{64\pi}\left[\frac{\left(1 - 4x_W + 8x_W^2\right)^2}{x_W^2(1 - x_W)^2}\right]\left[\frac{-9s + 12\sqrt{s}E_\gamma + 9E_\gamma^2\sin^2\theta_\gamma - 4\sqrt{s}E_\gamma\sin^2\theta_\gamma}{\sqrt{s}E_\gamma\sin^2\theta_\gamma\left[\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2\right]}\right]$$
$$\times\left[\frac{1}{\sqrt{3 - 4x_W}}\right]E_\gamma \mathrm{d}E_\gamma \mathrm{d}\cos\theta_\gamma, \tag{11}$$

$$E = \int \frac{\alpha^3}{16}\left[\frac{\left(1 - 4x_W + 8x_W^2\right)^2}{x_W^2(1 - x_W)^2}\right]$$
$$\times\left[\frac{-s^2 + 3s\sqrt{s}E_\gamma - 2sE_\gamma^2 + \frac{1}{2}sE_\gamma^2\sin^2\theta_\gamma - \frac{3}{4}\sqrt{s}E_\gamma^3\sin^2\theta_\gamma}{sE_\gamma^2\sin^2\theta_\gamma\left[\left(s - M_Z^2\right)^2 + M_Z^2\Gamma_Z^2\right]}\right]$$
$$\times E_\gamma \mathrm{d}E_\gamma \mathrm{d}\cos\theta_\gamma, \tag{12}$$

$$F = -\int \frac{\alpha^3}{4}\left[\frac{\left(1-4x_W+8x_W^2\right)^2}{x_W^2(1-x_W)^2}\right] \times \left[\frac{-s^2+3s\sqrt{s}E_\gamma-2sE_\gamma^2+\frac{1}{2}sE_\gamma^2\sin^2\theta_\gamma-\frac{3}{4}\sqrt{s}E_\gamma^3\sin^2\theta_\gamma}{sE_\gamma^2\sin^2\theta_\gamma\left[\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right]}\right] \times \left[\frac{1}{\sqrt{3-4x_W}}\right]E_\gamma \mathrm{d}E_\gamma \mathrm{d}\cos\theta_\gamma. \tag{13}$$

The expression given for A corresponds to the cross section previously reported by Grifols and Mendez [22], while B, C, D, E and F comes from the contribution of the 331 model, of the interference and the SM contribution due to bremsstrahlung in which the photon is radiated by the final tau or antitau. Evaluating the limit when the mixing angle is $\phi = 0$, the terms that depend on ϕ in (7) are zero and equation (7) is reduced to the expression (4) given in [22], more the contribution of the interference and the contribution of the SM, respectively.

In the case of the weak dipole moments, to get the expression for the differential cross section, we have to substitute the Z 331 model couplings given in equation (9) of [51] with the respective weak dipole moments included in equation (1), that is to say $a_\tau^W = F_2^Z(q^2 = m_Z^2)$ and $d_\tau^W = eF_3^Z(q^2 = m_Z^2)$. We do not reproduce the analytical expressions here because they are rather similar to the term given in equation (6). In the following section we will present the bounds obtained for the tau dipole moments using the data published by the L3 and OPAL Collaborations for the reaction $e^+e^- \to \tau^+\tau^-\gamma$ [1, 2].

3. Results

In practice, detector geometry imposes a cut on the photon polar angle with respect to the electron direction, and further cuts must be applied on the photon energy and minimum opening angle between the photon and tau in order to suppress background from tau decay products. In order to evaluate the integral of the total cross section as a function of the parameters of the 331 model, that is to say, ϕ, we require cuts on the photon angle and energy to avoid divergences when the integral is evaluated at the important intervals of each experiment. We apply the cuts used by L3 Collaboration [1] for the photon angle and energy, that is to say, we integrate over $\cos\theta_\gamma$ from -0.74 to 0.74 and E_γ from 5 to 45.5 GeV for various fixed values of the mixing angle $\phi = -3.979 \times 10^{-3}, 0, 1.309 \times 10^{-4}$. Using the following numerical values: $\sin^2\theta_W = 0.2314$, $M_Z = 91.18$ GeV, $\Gamma_Z = 2.49$ GeV and $m_\tau = 1.776$ GeV, we obtain the cross section $\sigma = \sigma(\phi, a_\tau, d_\tau)$.

For the mixing angle ϕ between Z and Z' of the 331 model, we use the reported data of Cogollo *et al* [51]:

$$-3.979 \times 10^{-3} \leqslant \phi \leqslant 1.309 \times 10^{-4}, \tag{14}$$

with a 90% C L. Other limits on the mixing angle ϕ reported in the literature are given in [48].

As was discussed in [27], $N \approx \sigma(\phi, a_\tau, d_\tau)\mathcal{L}$, where $N = 1559$ is the total number of $e^+e^- \to \tau^+\tau^-\gamma$ events expected and $\mathcal{L} = 100\ pb^{-1}$, according to the data reported by the L3 Collaboration [1] and references therein. Taking this into consideration, we can get a bound for the tau MM as a function of ϕ with $d_\tau = 0$. The values obtained for this bound for several values of the ϕ parameter are show in table 1. The previous analysis and comments can readily be translated to the EDM of the tau with $a_\tau = 0$. The resulting bounds for the EDM as a function of ϕ are shown in table 1. As expected, the limits obtained for the electromagnetic dipole moments of the tau-lepton are consistent with those obtained by these collaborations from the data for the process $e^+e^- \to \tau^+\tau^-\gamma$ [1, 2].

Table 1. Limits on the a_τ magnetic moment and d_τ electric dipole moment at 90 % C. L. for different values of the mixing angle ϕ. We have applied the cuts used by L3 for the photon angle and energy.

ϕ	a_τ	$d_\tau (10^{-16}\ e$ cm)
-3.979×10^{-3}	$[-0.049, 0.025]$	$[-2.72, 1.38]$
0	$[-0.052, 0.028]$	$[-2.88, 1.55]$
1.309×10^{-4}	$[-0.053, 0.029]$	$[-2.94, 1.61]$

Table 2. Limits on the a_τ^W anomalous weak magnetic moment and d_τ^W weak electric dipole moment at 90 % C L of the τ lepton for different values of the mixing angle ϕ. We have applied the cuts used by L3 for the photon angle and energy.

ϕ	$a_\tau^W (10^{-3})$	$d_\tau^W (10^{-17}\ e$ cm)
-3.979×10^{-3}	$[-2.166, 2.086]$	$[-1.202, 1.158]$
0	$[-2.193, 2.113]$	$[-1.217, 1.173]$
1.309×10^{-4}	$[-2.194, 2.114]$	$[-1.218, 1.174]$

The bounds for the weak dipole moments of the tau-lepton according to the data from the L3 Collaboration [1] for the energy and the opening angle of the photon, as well as the luminosity and the events numbers, are given in table 2. As we can see, the use of the strong limit obtained for the mixing angle ϕ of the 331 model also induces stringent bounds for the tau weak dipole moments, which are already consistent with those bound recently obtained by the DELPHI and ALEPH Collaborations in the process $e^+e^- \to \tau^+\tau^-$ [3, 4]. Our results in table 2 for $\phi = -3.979 \times 10^{-3}$, 0, 1.309×10^{-4} differ by about a factor of two of the bounds obtained by the DELPHI/ALEPH Collaborations for the tau weak dipole moments [3, 4, 6], our analysis is not sensitive to the real and imaginary parts of these parameters separately. The much more stringent limits on the weak anomalous moments reported in the literature [3–5, 27] are dominated by the reaction $Z \to \tau^+\tau^-$ with no photon, since there are many more events in that case, and the weak moments do not depend on the presence of a final state photon. In order to improve our limits it might be necessary to study direct CP-violation effects [55, 56].

We plot the total cross section in figure 2 as a function of the mixing angle ϕ of the 331 model for the bounds of the MM given in table 1. In figure 2, for $\phi = 1.309 \times 10^{-4}$ we reproduce the data previously reported in the literature. Our results for the dependence of the differential cross section on the photon energy versus the cosine of the opening angle between the photon and the beam direction (θ_γ) are presented in figure 3 for $\phi = 1.309 \times 10^{-4}$ and $a_\tau = -0.053$. We plot the differential cross section in figure 4 as a function of the photon energy for the bounds of the MMs given in table 1. We observe in this figure that the energy distributions are consistent with those reported in the literature. Our results for the dependence of the differential cross section on the photon energy versus the mixing angle ϕ of the model are presented in figure 5 for $a_\tau = -0.053$. In addition, the form of the distributions does not change significantly for ϕ because ϕ is very small in value, as shown in equation (14). In figure 6, for $\phi = 1.309 \times 10^{-4}$ we presented the results for the dependence of the differential cross section on the anomalous MM a_τ versus the cosine of the opening angle between the photon and the beam direction (θ_γ). From this figure is clear the dependence of the differential cross section with respect to the MM of the tau-lepton a_τ, as well as of the cosine of the opening angle $\cos\theta_\gamma$. Besides, we plot the differential cross section in figure 7 as a function of the

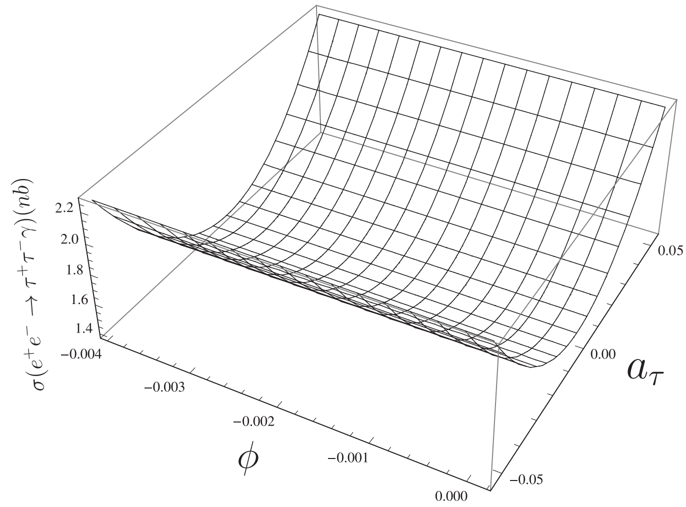

Figure 2. The total cross section for $e^+e^- \to \tau^+\tau^-\gamma$ as a function of ϕ and a_τ (table 1).

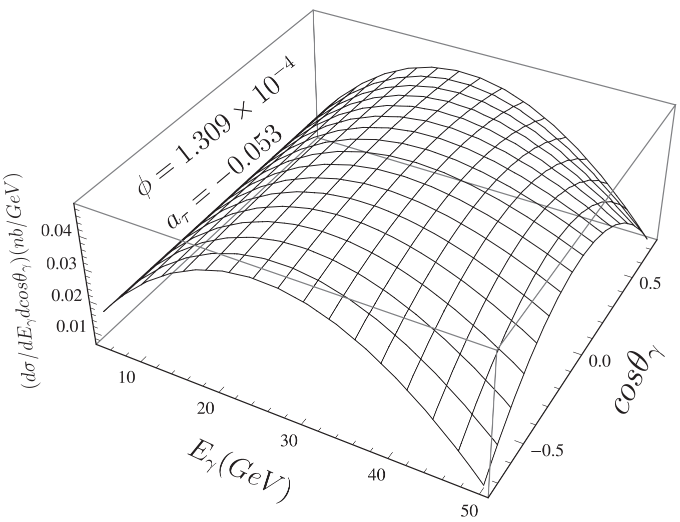

Figure 3. The differential cross section for $e^+e^- \to \tau^+\tau^-\gamma$ as a function of E_γ and $\cos\theta_\gamma$ for $\phi = 1.309 \times 10^{-4}$ and $a_\tau = -0.053$.

mixing angle ϕ and the cosine of the opening angle between the photon and the beam direction (θ_γ) with $a_\tau = -0.053$. We observed in this figure a small dependence of the differential cross section with respected to the mixing angle ϕ of the 331 model.

By reversing the process we determine a limit on the Z–Z' mixing angle from the expression of the scattering cross section of the process $e^+e^- \to \tau^+\tau^-\gamma$. The bounds on the mixing angle ϕ have been obtained by using the lower bounds on the anomalous contributions to the MM and EDM of tau, which maximizes the total cross section, namely $a_\tau = -0.052$. We show the dependence of the total cross section for the process $e^+e^- \to \tau^+\tau^-\gamma$ with

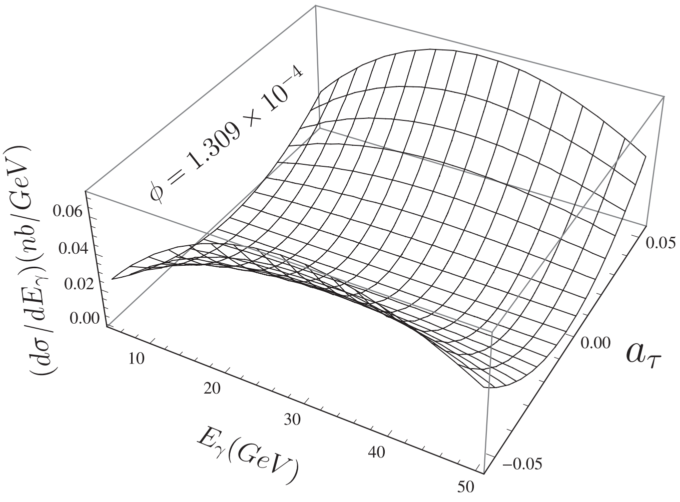

Figure 4. The differential cross section for $e^+e^- \to \tau^+\tau^-\gamma$ as a function of E_γ and a_τ for $\phi = 1.309 \times 10^{-4}$.

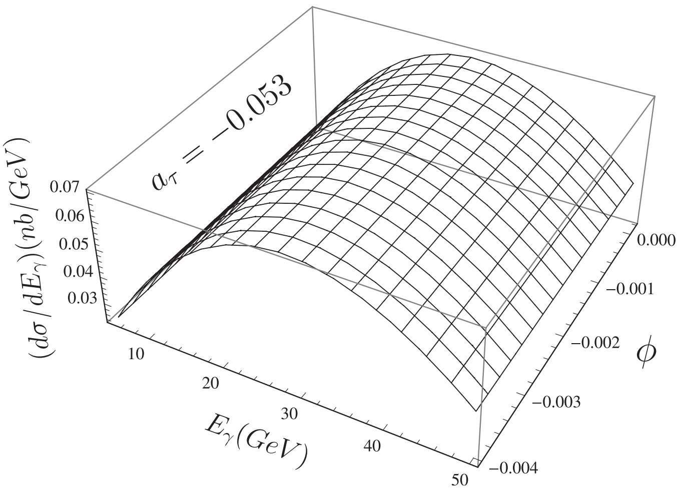

Figure 5. The differential cross section for $e^+e^- \to \tau^+\tau^-\gamma$ as a function of E_γ and ϕ with $a_\tau = -0.053$.

respect to the mixing angle ϕ between Z and Z' in figure 8. As an illustration we also include the cases where $a_\tau = 0.013$ is its upper limit and $a_\tau = 0.001$ for the value of the SM. Using the data $\sigma = (1.472 \pm 0.006 \pm 0.020)$ nb [1, 23], for the cross section, where the first error is statistical and the second is systematic, we get the following limits for ϕ:

$$-3.92 \times 10^{-3} \leqslant \phi \leqslant 1.30 \times 10^{-4}, \tag{15}$$

with a 90% C L which are consistent with those obtained recently from the LEP data on the number of light neutrino species in the 331 model [51]. The bound obtained in equation (15)

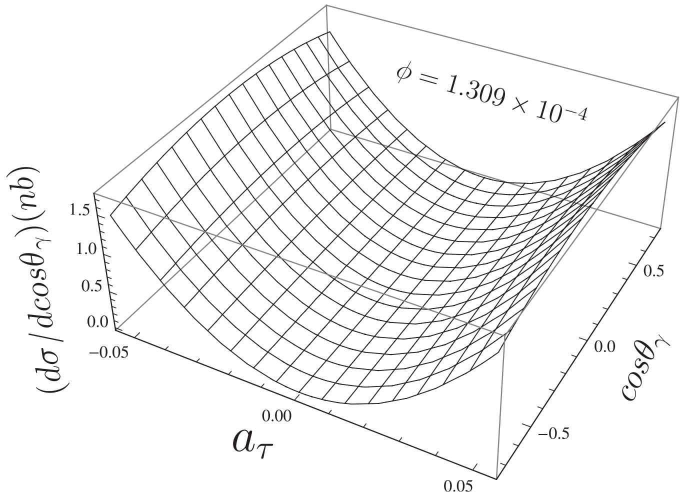

Figure 6. The differential cross section for $e^+e^- \rightarrow \tau^+\tau^-\gamma$ as a function of a_τ and $\cos\theta_\gamma$ with $\phi = 1.309 \times 10^{-4}$.

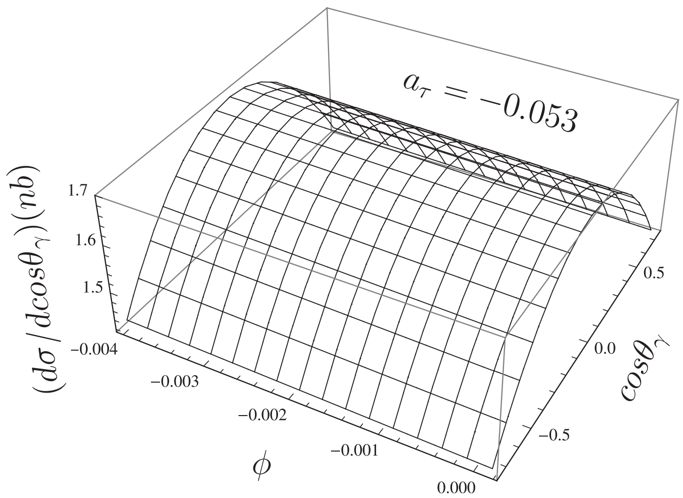

Figure 7. The differential cross section for $e^+e^- \rightarrow \tau^+\tau^-\gamma$ as a function of ϕ and $\cos\theta_\gamma$ with $a_\tau = -0.053$.

is very clean and does not depend on the mass of heavy neutral gauge boson Z'. Other limits on the mixing angle ϕ reported in the literature are given in [48, 52].

Finally, we find that the effects induced by the tree level Ze^+e^- and $Z\tau^+\tau^-$ couplings in the 331 model increase the cross section of the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$, and the predictions on the electromagnetic and weak dipole moments of the tau-lepton are better estimated. However, it is necessary to make an analysis at loop level for the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ in the context of a 331 model.

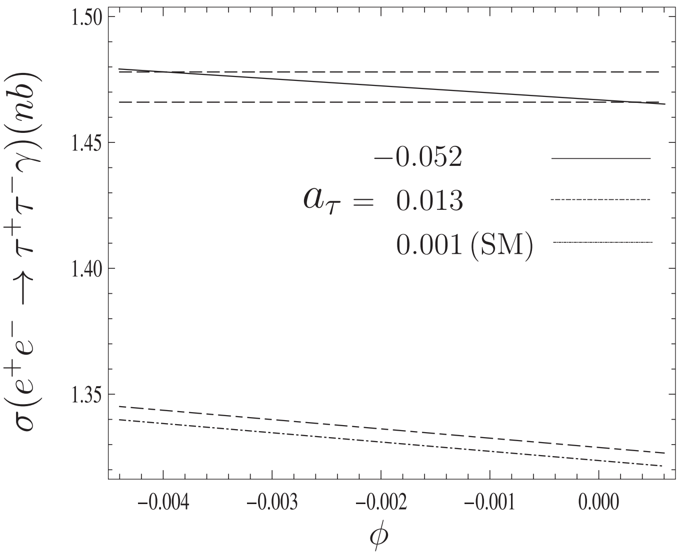

Figure 8. The curves show the shape for $e^+e^- \to \tau^+\tau^-\gamma$ as a function of the mixing angle ϕ and different values of the a_τ magnetic moment. Starting from the top, the curves are for $a_\tau = -0.052, 0.013, 0.001$ (SM).

4. Conclusions

From the total cross section for the reaction $e^+e^- \to \tau^+\tau^-\gamma$ at the Z pole and in the framework of a 331 model, we obtained bounds on the electromagnetic and weak dipole moments of the tau-lepton using the data published by the L3 and OPAL Collaborations for the process $e^+e^- \to \tau^+\tau^-\gamma$. We also obtained bounds on the tau weak dipole moments which are consistent with the bounds obtained recently by the DELPHI, ALEPH and BELLE Collaborations from the reaction $e^+e^- \to \tau^+\tau^-$ [3–5]. In addition, we get a limit on the Z–Z' mixing angle of the 331 model which is competitive with those reported in the literature [48, 51]. Our work complements other studies on the electromagnetic and weak dipole moments of the tau-lepton and on the Z–Z' mixing angle. In the limit $\phi = 0$, our bound take the value previously reported in [22] for the SM. As far as the weak dipole moments are concerned, our limits given in Tables 1 and 2 are consistent with the experimental bounds obtained at LEP with the two-body decay mode $Z \to \tau^+\tau^-$ [6]. On the other hand, it seems that in order to improve these bounds it might be necessary to study direct CP-violating effects [55, 56], as well as make an analysis at loop level for the process $e^+e^- \to \tau^+\tau^-\gamma$ in the context of a 331 model. In addition, the analytical and numerical results for the total cross section have never been reported in the literature before and could be of relevance for the scientific community.

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